Quantum dimer models

On the doping issue

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OUTLINE

- Some motivations & general consideration on Quantum Dimer Models
- Introducing the doping issue
- Generalizing the RK point at finite doping
  - Classical simulations
- Phase separation of the doped QM: Green function Quantum Monte Carlo

Collaborators

- Arnaud Ralko (Toulouse)
- Frédéric Mila (Lausanne)
- Federico Becca (SISSA)

Quantum DM

- Fabien Trousselet (Toulouse)
- Fabien Alet (Toulouse)
- Pierre Pujol (Toulouse)

Classical DM

D. Poilblanc et al., Phys. Rev. B 74, 014437 (2006)

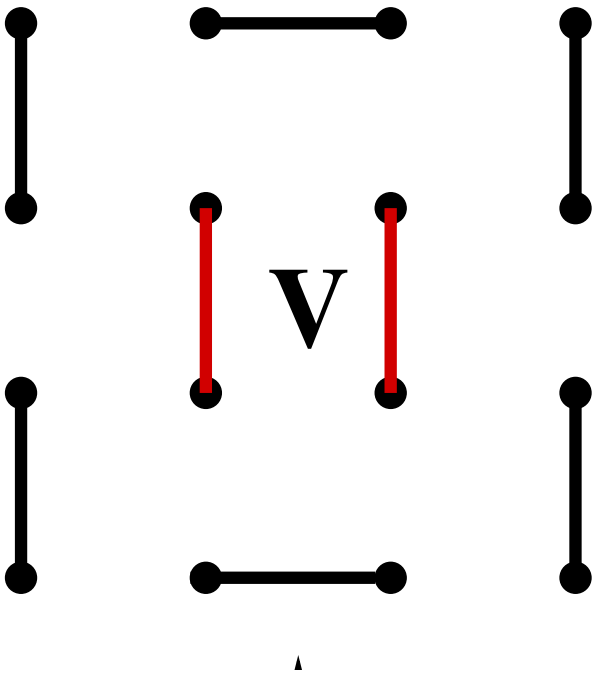
A. Ralko et al., in preparation → **cf. Poster**

F. Trousselet et al., in preparation → **cf. Poster**

Motivations

- Construct models to focus on low energy dynamics in the **singlet** sector
- Ignore magnetic excitations: justified for **gapped** magnons or spinons
- Need for models **simpler than spin models** but can nevertheless exhibit both **liquid-like and/or crystal-like ground states**

Classical dimer model

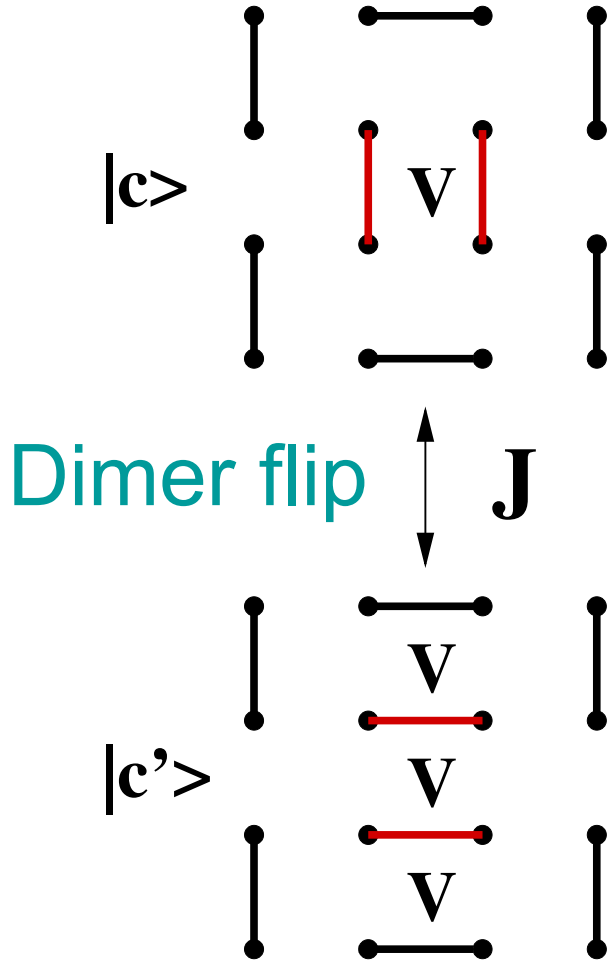


A typical (hard-core) dimer covering of the square lattice

Dimer interaction

$$E_{\text{clas}} = V N_c = e_c$$

Number of “V” plaquette in configuration |c>



Adding quantum fluctuations:

The Quantum Dimer Model

Rokhsar & Kivelson, PRL 88

$$H_{\mathrm{QDM}} = \sum_c e_c |c\rangle\langle c| - J \sum_{c,c'} |c\rangle\langle c'|$$

Relation with SU(2) spin models

SU(2) Valence Bond ⇔ dimer covering

Orthogonal basis by construction

Sutherland, 1988:

$$|\langle a|b\rangle| = 2^{(n_{\mathcal{L}} - N/2)} = \prod_{\mathcal{L}} 2^{(1 - L_{\mathcal{L}}/2)}$$

Length of the loops of overlap graph

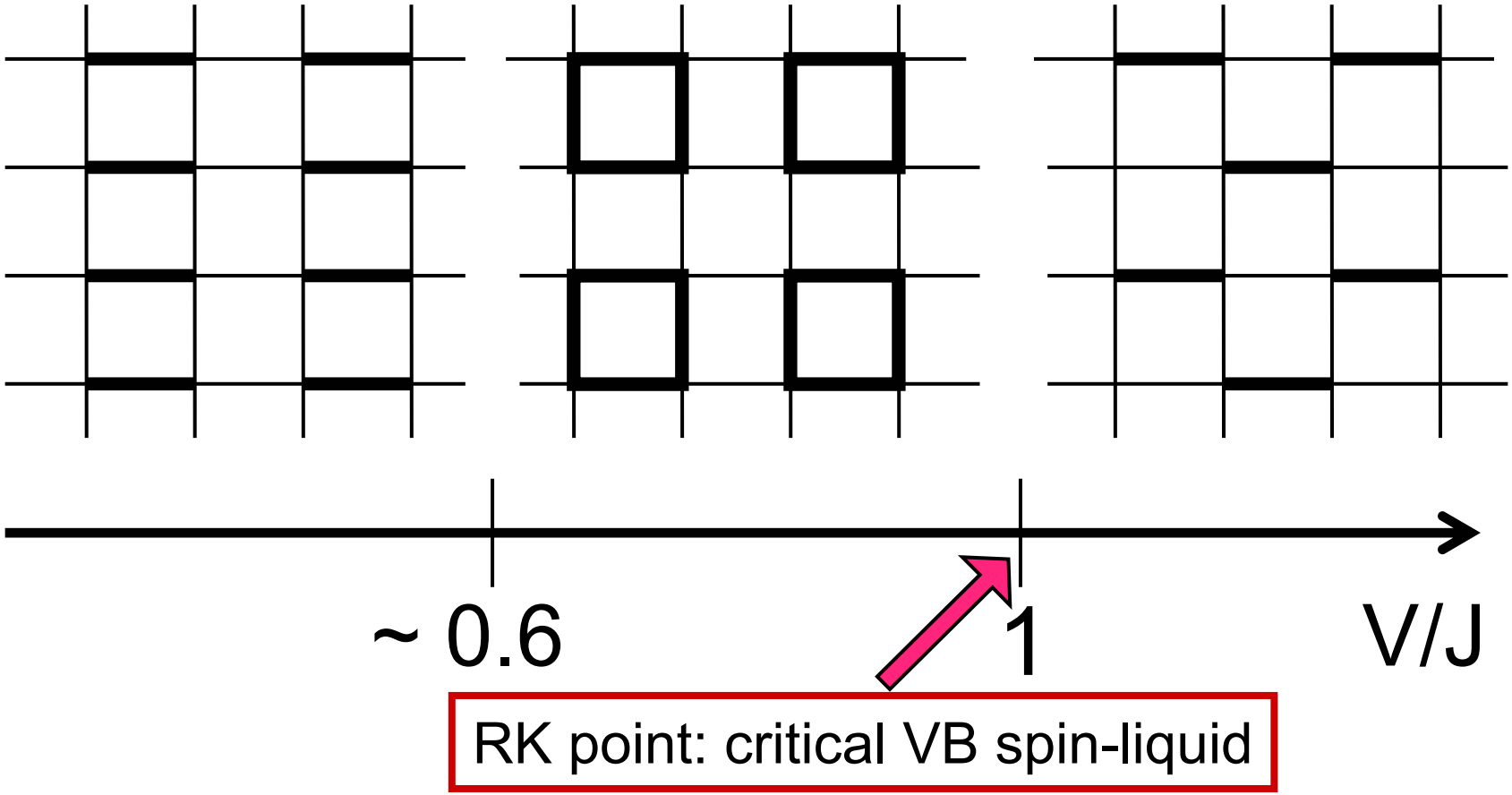
Small parameter : $\left(\frac{1}{\sqrt{2}}\right)^{L_{\mathcal{L}}}$

RK, 1988: Expansion to order x^n

→ Hamiltonian with up to n-dimers terms

Phase diagram

No minus sign problem => QMC: Syljuasen, PRB 2006



Rokhsar-Kivelson point

For J=V: sum of projectors

$$H_{\mathrm{RK}} = \sum_{\mathrm{p}} |\Psi_p\rangle\langle\Psi_p|$$

$$|\Psi_{c,c'}\rangle = \frac{1}{\sqrt{2}}(|c\rangle - |c'\rangle)$$

$$|\Phi_0\rangle = \frac{1}{Z}\sum_{\{c\}} |c\rangle$$

exact GS with energy E=0

$\Leftrightarrow$ Infinite-T Classical DM

Quasi-long ranged (critical) dimer-dimer correlations

RVB liquid on the triangular lattice

Moessner & Sondhi, PRL 2001

Dimer flips on all Rhombi (3 kinds) of the lattice

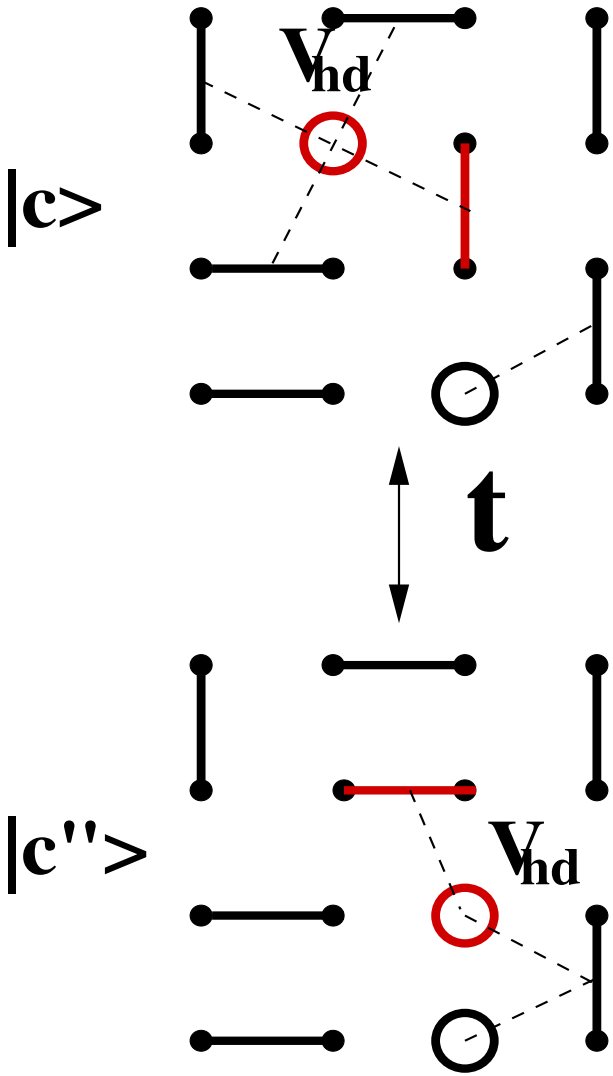
Again for V=J, mapping to classical problem (RK point)

Pfaffian calculation

Finite correlation length
Exponential decay of dimer-dimer correlations

Degeneracy from “topological order”
GS have different “winding numbers”

Motion of “holons”



Inject vacancies (by pairs) & add new term in H:

$$-t \sum_{c,c''} |c\rangle\langle c''|$$

Generalized RK points: recipe

Following Castelnovo et al., Annals of Physics **318**, 316, 2005

$$e_c = e_c^{\text{flip}}(\beta) + e_c^{\text{hop}}(\beta)$$

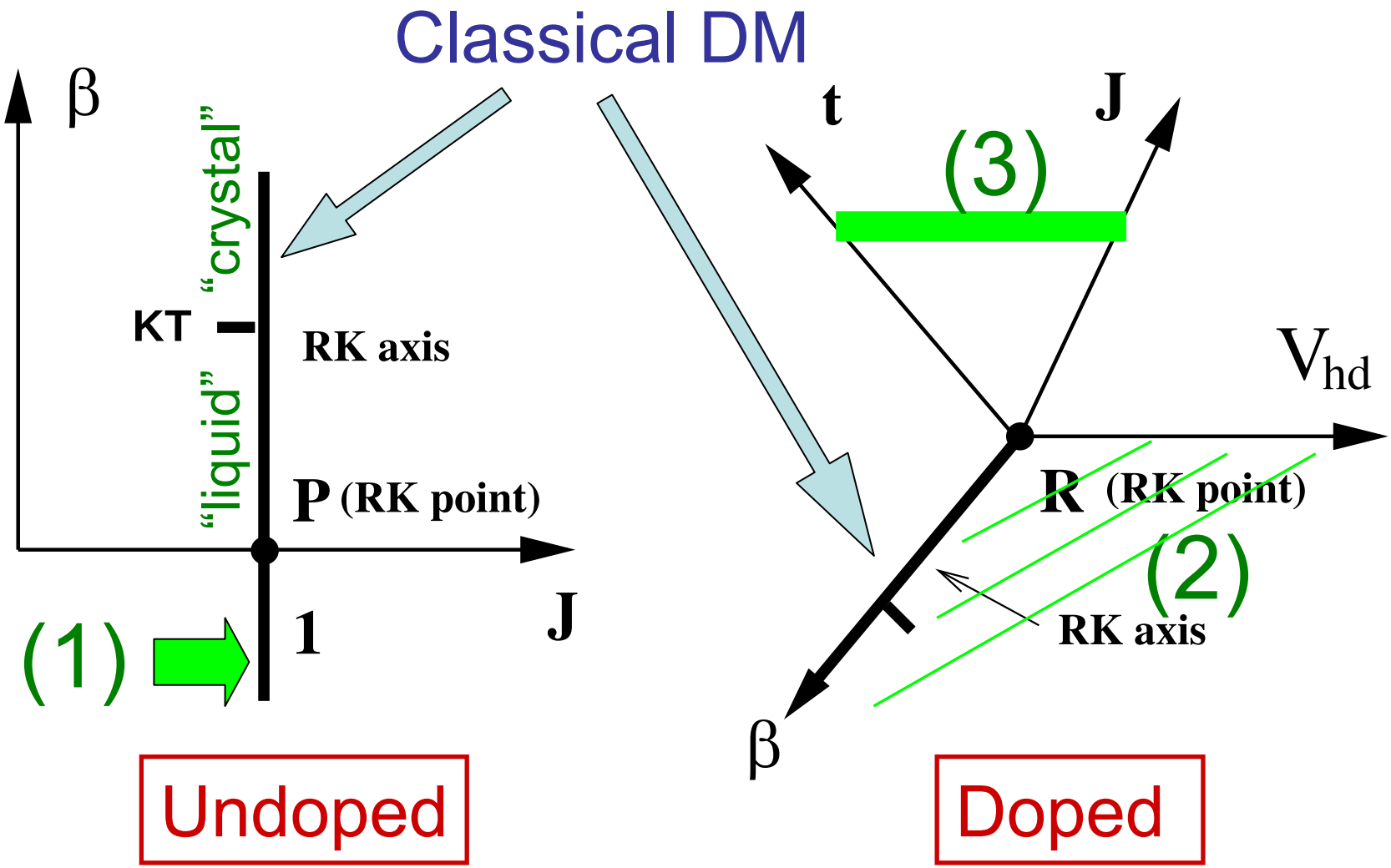
$$\epsilon_c^{\text{flip}} = V \sum_{c'(c)} \exp\{-\frac{1}{2}\beta V_{\text{cl}}(N_{c'} - N_c)\}$$

$$\epsilon_c^{\text{hop}} = V_{\text{hd}} \sum_{c''(c)} \exp\{-\frac{1}{2}\beta V_{\text{cl}}(N_{c''} - N_c)\}$$

$$V_{\text{cl}} = \pm 1$$

For V=J and V_{hd}=t :

$$|\Phi_0\rangle = \frac{1}{\sqrt{Z}} \Sigma_{\{c\}} \exp\left(-\frac{\beta}{2} V_{\text{cl}} N_c\right) |c\rangle$$



Generalized QDM's

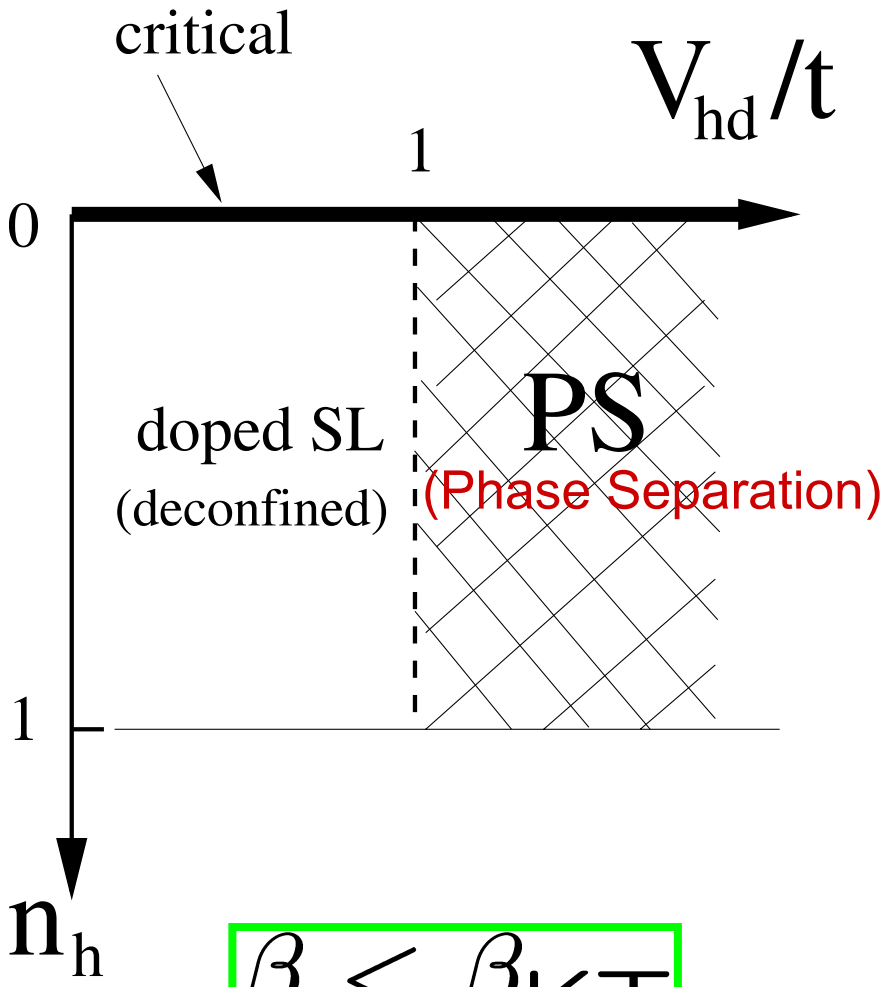
(1), (2) and (3) projects

- (1) Classical DM: from the square to the triangular lattice
  - F. Alet et al. PRL 2006 (square lattice)
  - F. Trousselet, F. Alet, P. Pujol, D.P. (transfer matrix techniques**, cf. Poster**)

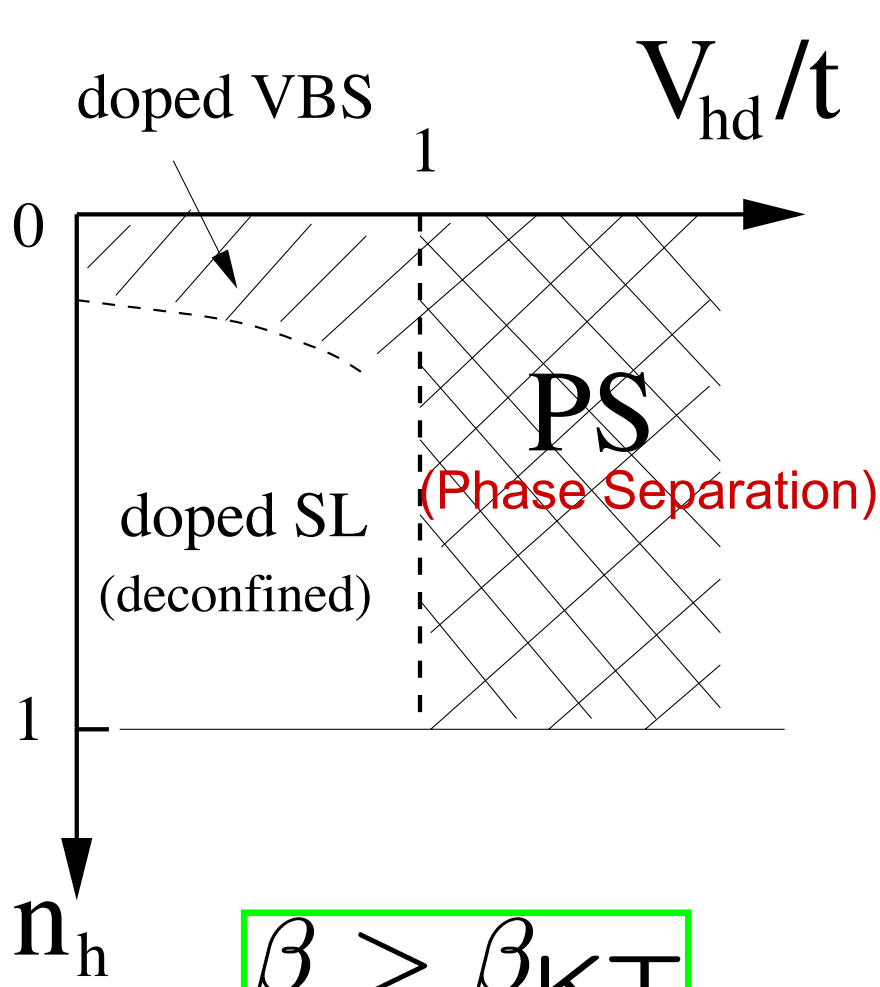
- (2) V=J: $t - V_{\mathrm{hd}} - \beta$ phase diagram (PRB 2006)

- (3) "Conventional" t-J QDM: $V_{hd}=0$ and $\beta = 0$
  - A. Ralko, F. Mila, D.P. **(cf. Poster)**

(2)



$\beta < \beta_{KT}$

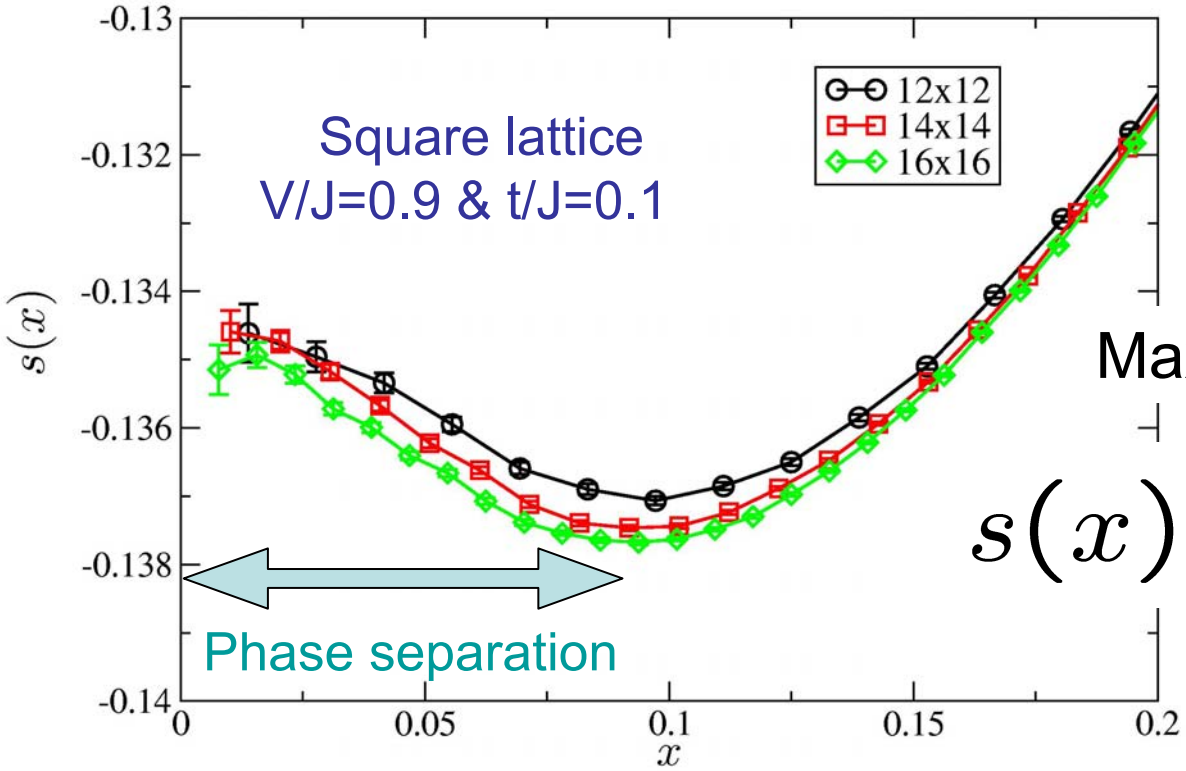


$\beta > \beta_{KT}$

n_h=doping ratio

(3)

Green function Monte Carlo computations



doping ratio

Maxwell construction:

$$s(x) = \frac{e(x) - e(0)}{x}$$

See A. Ralko poster: nature of metallic phase?
triangular vs square lattice ?
etc…

Summary / Conclusions

- Frustration + quantum fluctuations lead to exotic disordered GS (VBC, SL, …)
- Possible realization of exotic physics (deconfined spinons, Deconfined Critical Points, etc…)
- Variety of fascinating materials (insulators) to look for such behaviors (pyrochlores, Kagome, etc…)
- Microscopic models are hard to simulate (Exact diagonalisations) but effective QDM easier
- The doping issue might reserve many surprises but needs further investigations

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