Gaz de Van Der Waals : calcul du point critique (v_c, T_c, P_c)

$$P = \frac{k_B T}{v-b} - \frac{a}{v^2} \quad (1)$$

Point critique $\Rightarrow$ $\frac{\partial P_c}{\partial v_c} = 0$, $\frac{\partial^2 P_c}{\partial v_c^2} = 0$

$$\frac{\partial P_c}{\partial v_c} = \frac{-k_B T_c}{(v_c-b)^2} + \frac{2a}{v_c^3} = 0 \iff \frac{k_B T_c}{(v_c-b)^2} = \frac{2a}{v_c^3} \quad (2)$$

$$\frac{\partial^2 P_c}{\partial v_c^2} = \frac{2 k_B T_c}{(v_c-b)^3} - \frac{6a}{v_c^4} = 0 \iff \frac{2 k_B T_c}{(v_c-b)^3} = \frac{6a}{v_c^4} \quad (3)$$

$$(3)/(2) \rightarrow \frac{2 k_B T_c}{(v_c-b)^3} \Big/ \frac{k_B T_c}{(v_c-b)^2} = \frac{6a}{v_c^4} \Big/ \frac{2a}{v_c^3}$$

$$\iff \frac{2}{v_c-b} = \frac{3}{v_c} \iff 2v_c = 3(v_c-b)$$

donc $\boxed{v_c = 3b}$. On injecte dans (2)

$$\frac{k_B T_c}{(3b-b)^2} = \frac{2a}{27b^3} \rightarrow \boxed{T_c = \frac{2a\,4b^2}{27b^3 k_B} = \frac{8a}{27b\,k_B}}$$

$$(1) \rightarrow P_c = \frac{k_B \frac{8a}{27b\,k_B}}{3b-b} - \frac{a}{9b^2} = \frac{8a}{2b \times 27b} - \frac{a}{9b^2}$$

$$= \frac{4a}{27b^2} - \frac{3a}{27b^2} = \boxed{\frac{a}{27b^2} = P_c}$$