

Tutorial No3

Frustrated Heisenberg chain Deconfinement of spinons

Adapted from Sorensen et al., PRB 58, R14701 (1998)

Frustrated chain at MG point

- Let us consider a J1-J2 chain at the Majumdar-Ghosh point:

$$\alpha = J_2/J_1 = \frac{1}{2}$$

- Check the GS energy/site of L-site rings with L=2p even is:

$$E_0(L)/L = -\frac{3}{8}J$$

- What is the energy for momentum $k=\pi$?
Conclusion.

Soliton (kink) dispersion

- Interpolate the GS energy between $L=2p$ & $L=2p+2$ chains for odd “effective” # of sites:

$$E_0^*(2p + 1) = \frac{1}{2}(E_0(2p) + E_0(2p + 2))$$

- By considering chains of $L=2p+1$ sites, compute the kink dispersion:

$$e_S(k) = E_0(2p + 1, k) - E_0^*(2p + 1)$$

- Compare with Shastry & Sutherland variational ansatz (PRL 47, 964 (1981)):

$$e_V(k) = (J/2)(5/4 + \cos(2k))$$

Soliton-antisoliton binding energy

- Calculate the spin gap of $L=2p$ sites chains.
- What is the **physical interpretation** of such a triplet excitation ?
- Compute the soliton-antisoliton binding energy:

$$E_{\text{binding}} = \Delta^{01} - 2e_S^*$$

where e_S^* is the minimum soliton energy

Compare results with those of the lecture !