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Anisotropic Systems of Fermions: Gravitational Equilibrium and Dynamic Stability

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Introduction

- ***The history of fermions in astrophysics started at the beginning of XX century, by using this kind of particles to describe the final phases of the evolutive path of the stars (Fowler, 1926; Chandrasekhar, 1931; Oppenheimer & Volkoff, 1939).***

- ***Successively, fermions were connected to the dark matter problem both in Newtonian gravity (Cowsik & McClelland, 1972; Tremaine & Gunn, 1979) and in General Relativity (Bilic & Viollier, 1999).***

- ***In this talk we will consider systems of self – gravitating fermions with anisotropy in the distribution of velocities.***

Distribution Function

For the calculations: l = 1

$$
\begin{cases}
f = \dfrac{g}{h^3}\left(1+\dfrac{L^2}{L_c^2}\right)^{l} \dfrac{1-e^{(E-E_c)/k_B T}}{e^{(E-\mu)/k_B T}+1} & E \le E_c \\
f = 0 & E > E_c
\end{cases}
$$

Anisotropy Parameter: $a = r_a / \xi$

Values Chosen:
1, 0.5, 10^{-1}, 10^{-3}, 10^{-5}

$$
\vartheta = \frac{\mu}{k_B T}, \qquad W = \frac{E_c}{k_B T} = \frac{m(\Phi_R - \Phi)}{k_B T} = \vartheta - \vartheta_R
$$

Equations for the Gravitational Equilibrium

$$\frac{d^2W}{dr^2} + \frac{2}{r}\frac{dW}{dr} = -\frac{8\pi G}{\sigma^2}\rho$$

Newtonian

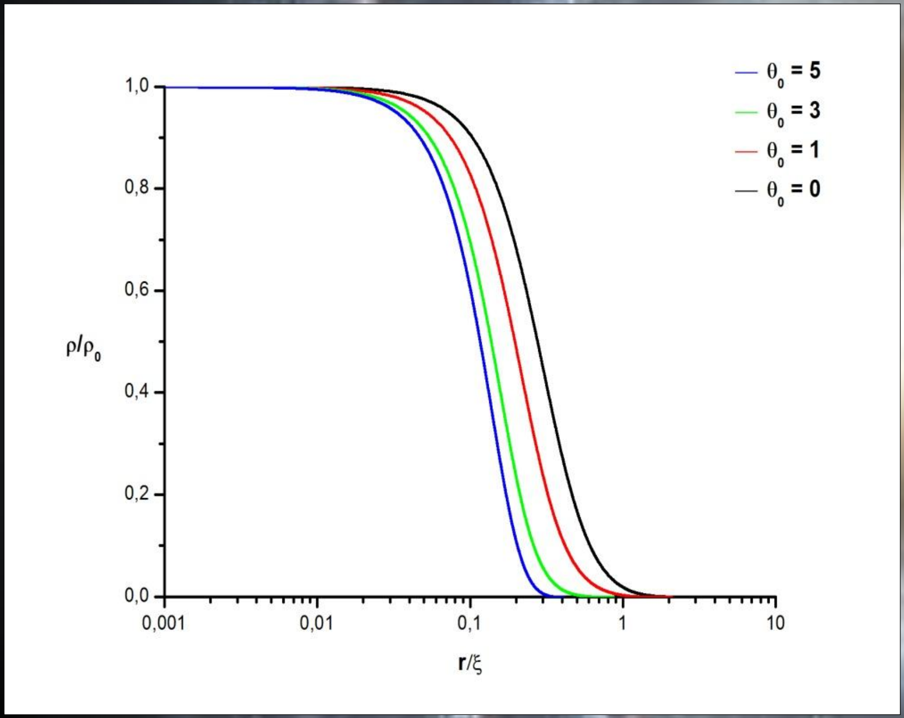
$$\begin{cases} \dfrac{dW}{dr} = -\dfrac{G}{c^2}\dfrac{1-\beta W}{\beta}\dfrac{M_r c^2 + 4\pi P_{rr} r^3}{r(rc^2 - 2GM_r)} \\ \dfrac{dM_r}{dr} = 4\pi\rho(r) r^2 \end{cases}$$

Relativistic

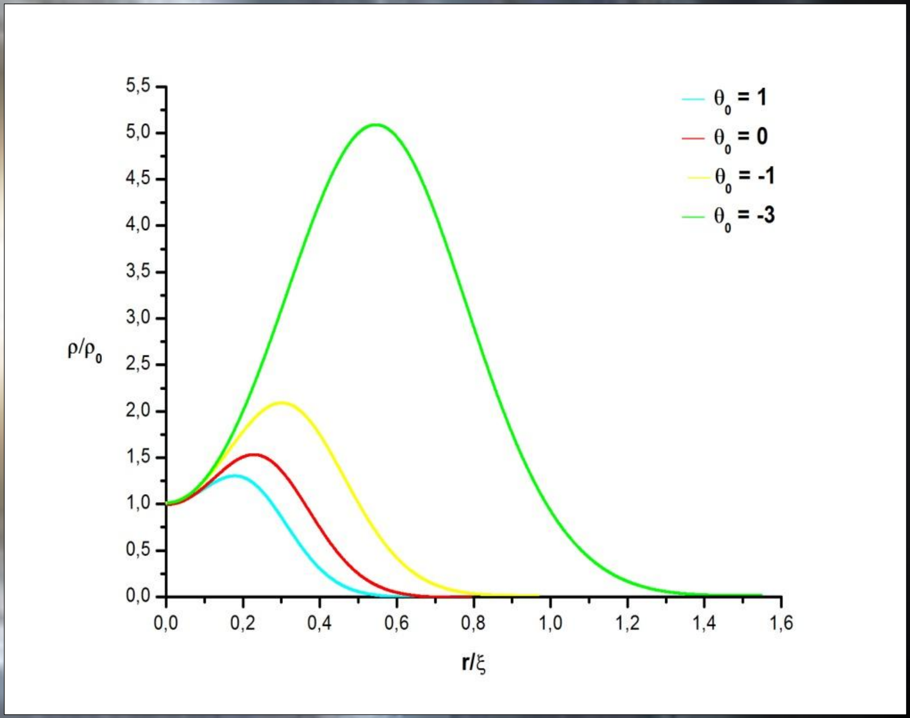
$$\beta = \frac{k_B T_R}{mc^2}$$

Newtonian Regime: Results

a = 0.5, W_0 = 7



a = 0.1, W_0 = 3

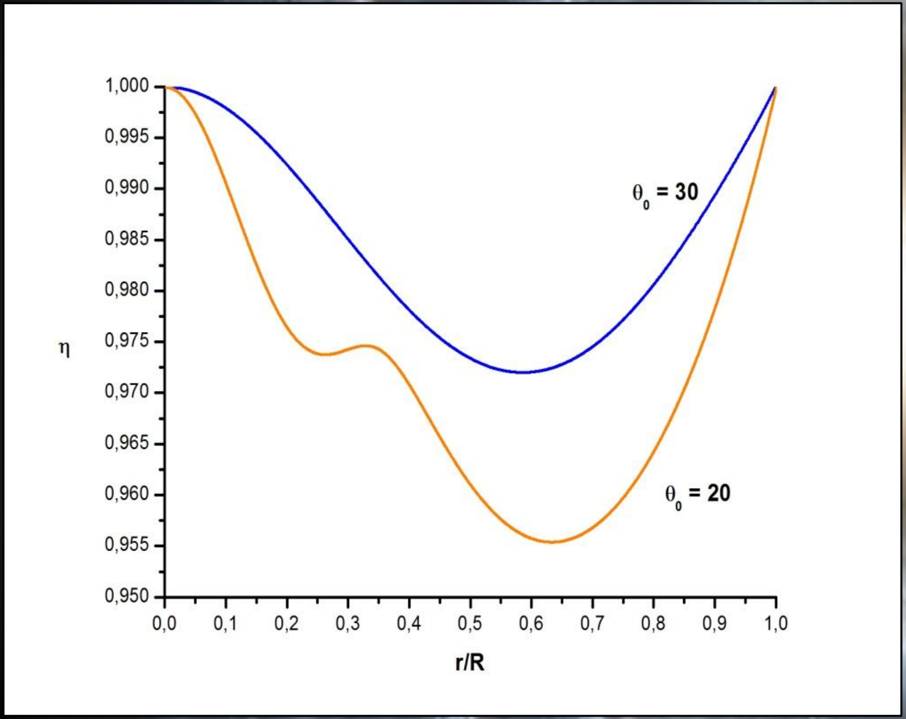


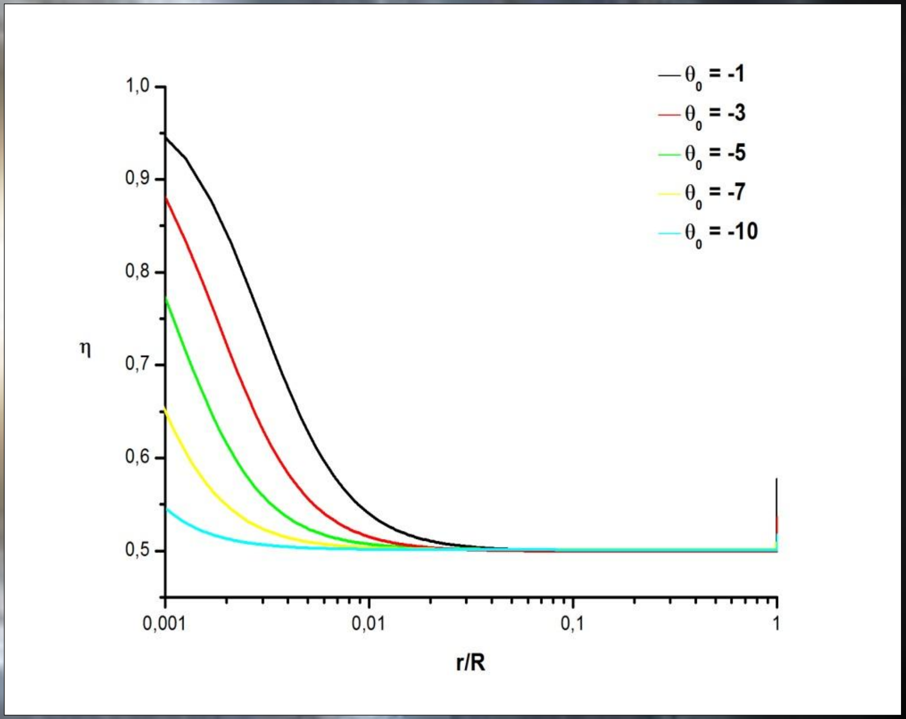
In the isotropic limit (left panel), the density profile recovers the trend of Ruffini & Stella (1983). For increasing anisotropy (right panel), we get the trend of "hollow systems" (Ralston & Smith 1991; Nguyen & Pedraza 2013).

Newtonian Regime: Results

$a = 1, W_0 = 30$

$a = 10^{-3}, W_0 = 10$

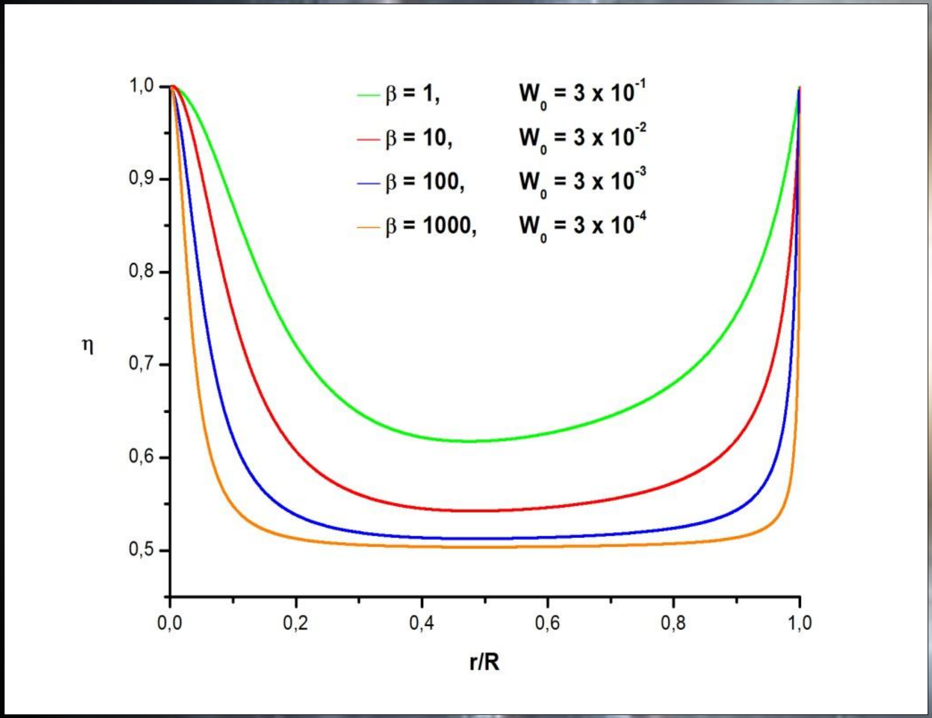




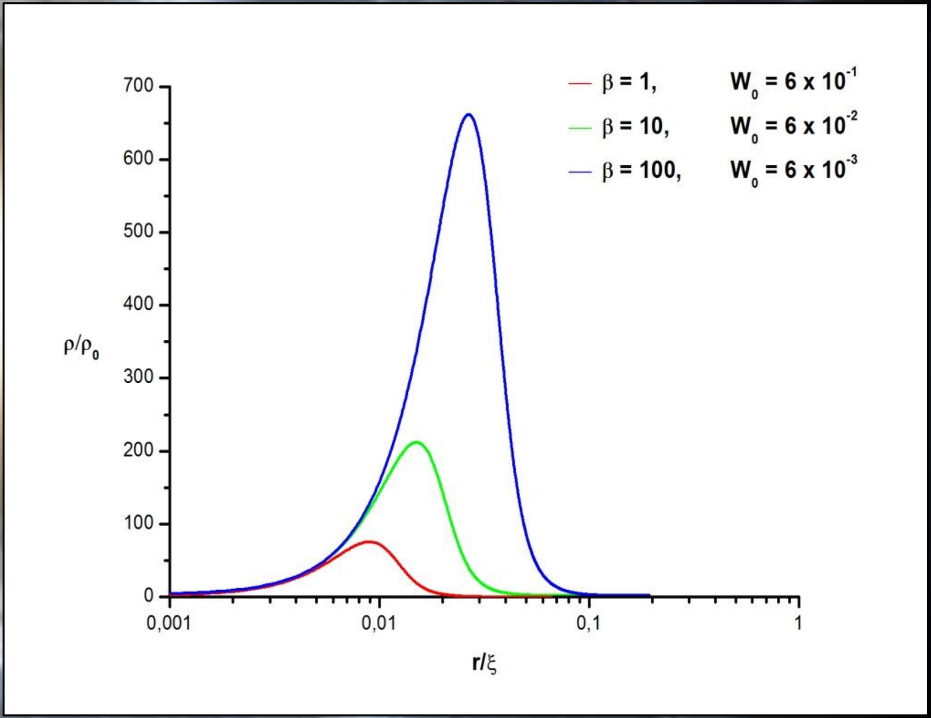
$\eta = P_{rr} / P_t$ (anisotropy level) is a gauge of the kind of orbits. Values of η close to 1 indicate the prevalence of a radial motion, values close to 0.5 a tangential one. See also Bisnovatyi – Kogan et al. (2009).

Relativistic Regime: Results

a = 10^−1^



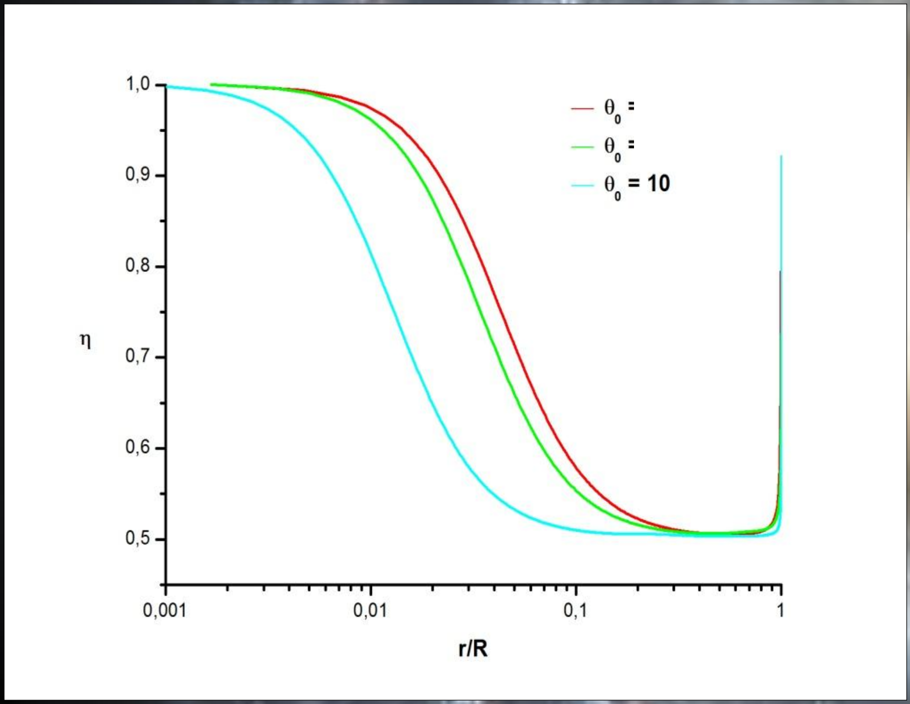
a = 10^−3^



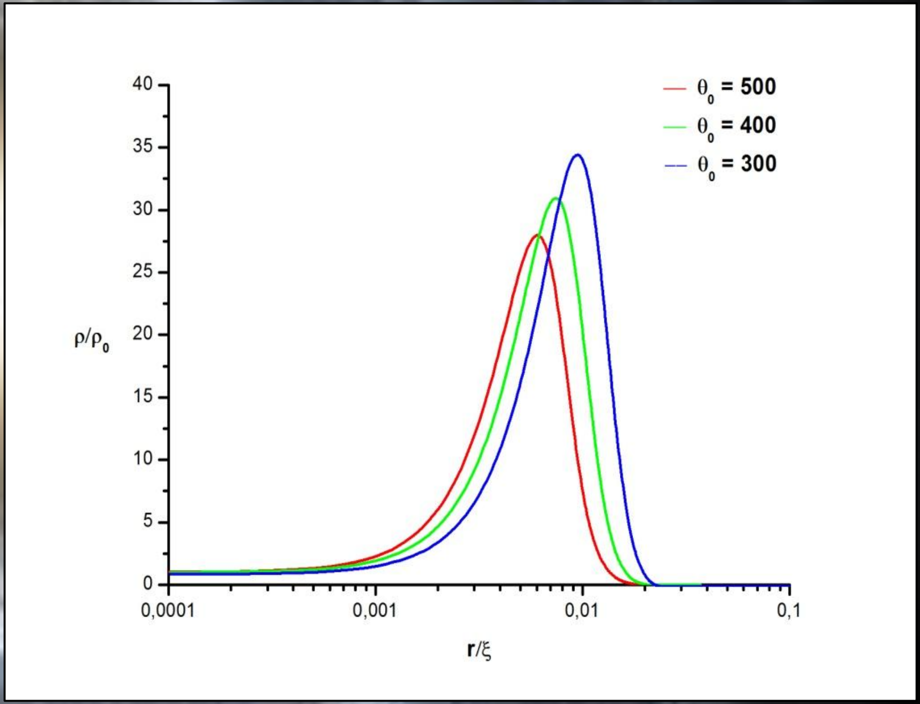
Behavior of η (left panel) and of density profile (right panel) by varying β (we have taken configurations with the same level of degeneracy). For more details about the meaning of β see Merafina & Ruffini (1989).

Relativistic Regime: Results

***a* = 10^{-3}, W_0 = 20, *β* = 10^{-2}**



***a* = 10^{-3}, W_0 = 500, *β* = 10^{-2}**



Behavior of η (left panel) and of the density profile (right panel) for fixed value of β. The profile shows again evidences for "hollowness" while η recovers the (classical) result of Bisnovatyi – Kogan et al. (2010).

Limits on Particles Mass (Newtonian)

From the distribution function, in the limit of full degeneracy ($\theta \to W$, $W \to \infty$), it is possible to derive an expression in which the mass is explicitly related to the anisotropy radius r_a .

$$m^4 \geq \frac{3\rho_c h^3}{4\pi g\sigma^3 W_c^{3/2}} \frac{1}{1+\frac{2}{5}W_c \frac{r_c^2}{r_a^2}}$$

W_c and ρ_c are the values of W and ρ at the core radius r_c .

Values obtained, for $\rho_c = (10^{-26} \div 10^{-17})$ g/cm^3 and $\sigma = 100$ km/s : $m \geq (10^{-3} \div 10^{3})$ eV.

Limits on Particles Mass (Relativistic)

$$m^4 \geq \frac{\rho_c h^3 \alpha_c^4}{2\pi g c^3 \beta^3} \times$$

$$\times \frac{1}{\sqrt{1-\alpha_c^2}\left(2-\alpha_c^2+\frac{8-14\alpha_c^2+3\alpha_c^4}{9\alpha_c^2 r_a^2}\beta^2 r_c^2\right)+\frac{\alpha_c^4}{3}\log\left(\frac{1+\sqrt{1-\alpha_c^2}}{\alpha_c}\right)\left(\frac{\beta^2 r_c^2}{r_a^2}-3\right)}$$

W_c , ρ_c and α_c (= 1 – βW_c) are the values of W, ρ and α at the core radius r_c .

Values obtained, for ρ_c = ($10^9 \div 10^{18}$) g/cm^3 : m ≥ (1 ÷ 10^4) GeV.

Dynamic Stability for Anisotropic Systems: Pulsation Equation for the Newtonian Case

General expression, valid for any type of fluid.

Lagrangian variation of a quantity
$\Delta Q = Q(x + \xi, t) - Q_0(x, t)$

$$\ddot{\xi} + \frac{4\xi P'_{rr}}{r\rho} - \frac{1}{\rho}\left[\frac{\gamma P_{rr}}{r^2}(r^2\xi)'\right]' + \frac{6\Pi\xi}{r^2\rho} + \frac{2\Pi\xi'}{r\rho} + \frac{2\Delta\Pi}{r\rho} = 0$$

Lagrangian Displacement

Adiabatic Index of the Perturbations

$$\gamma = \frac{d\ln P_{rr}}{d\ln\rho}$$

$\Pi = P_{rr} - P_t$

Dynamic Stability for Anisotropic Systems: Stability Criterion for the Newtonian Case

$$\langle \gamma \rangle \geq \gamma_{cr}$$

$$\langle \gamma \rangle = \frac{\int_0^R \gamma P_{rr} r^2 dr}{\int_0^R P_{rr} r^2 dr}$$

$$\gamma_{cr} = \frac{4}{9} + \frac{8}{9} \frac{\int_0^R P_t r^2 dr}{\int_0^R P_{rr} r^2 dr} - \frac{2}{9} \frac{\int_0^R (\partial_r \Pi) r^3 dr}{\int_0^R P_{rr} r^2 dr} + \frac{2}{3} \frac{\int_0^R (\partial_\rho \Pi) \rho r^2 dr}{\int_0^R P_{rr} r^2 dr}$$

Application of the Stability Criterion to the Systems of Fermions

		$a = 1$			$a = 0.5$		
W_0	θ_0	$<\gamma>$	γ_{cr}		$<\gamma>$	γ_{cr}	
50	50	1.6990	1.3391	S	1.8236	1.3534	S
	40	1.6833	1.3387	S	1.7928	1.3521	S
	30	1.6670	1.3381	S	1.7593	1.3502	S
30	30	1.6836	1.3378	S	1.7747	1.3495	S
	20	1.6313	1.3375	S	1.7039	1.3483	S
20	20	1.6665	1.3371	S	1.7380	1.3471	S
	15	1.6027	1.3371	S	1.6636	1.3471	S
15	15	1.6493	1.3366	S	1.7094	1.3455	S
	10	1.5294	1.3370	S	1.5767	1.3469	S
	7	1.3946	1.3413	S	1.4279	1.3546	S
10	10	1.6140	1.3362	S	1.6611	1.3440	S
	7	1.4988	1.3367	S	1.5386	1.3459	S
	5	1.3602	1.3386	S	1.3912	1.3515	S

γ_{cr} if $\Pi = \Pi(r)$ only →

$$\gamma_{cr} = \frac{10}{9} + \frac{2}{9}\frac{\int_0^R P_t r^2 dr}{\int_0^R P_{rr} r^2 dr} + \frac{2}{9}\frac{P_t(R)R^3}{\int_0^R P_{rr} r^2 dr}$$

Application of the Stability Criterion to the Systems of Fermions

		$a = 10^{-5}$			$a = 10^{-3}$			$a = 10^{-1}$		
W_0	θ_0	$< \gamma >$	γ_{cr}		$< \gamma >$	γ_{cr}		$< \gamma >$	γ_{cr}	
50	50	3.0063	1.5555	S	2.0585	1.5541	S	1.7991	1.4561	S
	40	-0.3757	1.5555	U	-6.2924	1.5541	U	1.0321	1.4533	U
	30	1.0578	1.5555	U	0.6432	1.5540	U	3.0525	1.4489	S
30	30	-0.3700	1.5555	U	-27.378	1.5540	U	4.8929	1.4470	S
	20	1.3953	1.5555	U	1.2194	1.5539	U	-3.1410	1.4435	U
	15	-2.3338	1.5555	U	3.0222	1.5539	S	1.4768	1.4409	S
20	20	-74.756	1.5555	U	2.0439	1.5539	S	1.8781	1.4398	S
	15	-0.1924	1.5555	U	6.0953	1.5539	S	1.5494	1.4398	S
	10	1.3922	1.5555	U	1.1854	1.5539	U	0.5352	1.4417	U
15	15	1.1374	1.5555	U	0.8224	1.5538	U	-0.3130	1.4349	U
	10	1.6042	1.5555	S	1.3209	1.5538	U	0.0075	1.4386	U
	7	1.3171	1.5555	U	1.0951	1.5540	U	1.5986	1.4476	S
	5	1.5214	1.5555	U	1.1974	1.5542	U	2.7489	1.4730	S
10	10	62.639	1.5555	S	1.6831	1.5536	S	1.8343	1.4290	S
	7	1.0195	1.5555	U	0.2248	1.5538	U	1.2211	1.4355	U
	5	0.8596	1.5555	U	1.6420	1.5540	S	1.4206	1.4481	U
	3	0.4099	1.5555	U	1.4389	1.5544	U	1.2455	1.4757	U
	1	1.1418	1.5555	U	1.1161	1.5550	U	1.1549	1.5117	U

Conclusions and Future Perspectives

We can divide the configurations in three zones:

1. *An internal region in which the motion is isotropic,*
2. *An intermediate region in which anisotropy prevails,*
3. *A region of frontier where the motion is isotropic.*

The behavior of density profiles shows the existence of hollow systems by indicating that the presence of the anisotropy is the "source" of this kind of configurations.

Possible applications: dSph Galaxies, Neutron Stars.

1. *Next step: extension of stability criterion to General Relativity,*
2. *Next step: study of the thermodynamic stability.*

For more details see Merafina & Alberti (2014), arXiv: 1402.0756

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