Supplementary material for "Projected wave function study of $\mathbb{Z}_2$ spin liquids on the Kagomé lattice for spin-$\frac{1}{2}$ quantum Heisenberg antiferromagnet"

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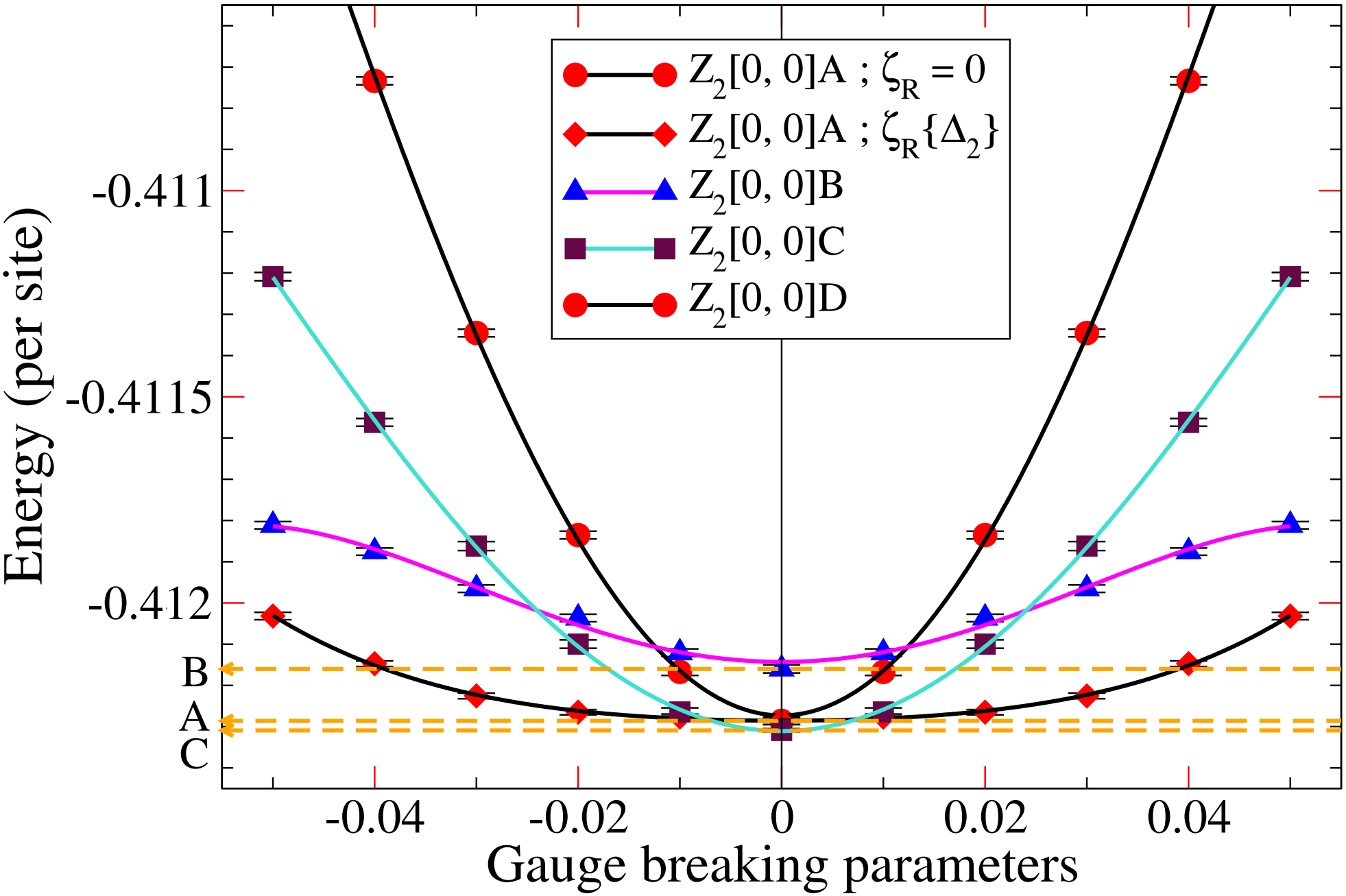
In this supplementary material, we present the explicit plots showing the variation of energy as a function of the $U(1) \to \mathbb{Z}_2$ gauge breaking parameters for the four gapped $\mathbb{Z}_2$ SLs in the neighborhood of the gapless $[0,0]$ SL (the uniform RVB state). We also reproduce the *ansatz* (from Table I of Ref. 21) of the five SLs investigated in paper, so as to make the paper self contained.

State	Λ_{onsite}	$U_{\text{n.n.}}$	$U_{2nd\text{n.n.}}$	$U_{3rd\text{n.n.}}$	$\tilde{U}_{3rd\text{n.n.}}$
$\mathbb{Z}_2[0,\pi]\beta$	μ, ζ_{R}	χ_{R}	$\chi_{\text{R}}, \Delta_{\text{R}}$	0	0
$\mathbb{Z}_2[0,0]$A	μ, ζ_{R}	χ_{R}	$\chi_{\text{R}}, \Delta_{\text{R}}$	0	0
$\mathbb{Z}_2[0,0]$B	μ	$\chi_{\text{R}}, \Delta_{\text{I}}$	0	0	0
$\mathbb{Z}_2[0,0]$C	μ	χ_{R}	χ_{R}	$\chi_{\text{R}}, \Delta_{\text{I}}$	χ_{R}
$\mathbb{Z}_2[0,0]$D	μ	χ_{R}	$\chi_{\text{R}}, \Delta_{\text{I}}$	0	0

TABLE I. The mean field *ansatz* of the five gapped SLs investigated by us, given only up to the neighbor at which the gauge symmetry is broken, in a form used by us in numerical simulations. The parameters highlighted in red are responsible for opening a gap by breaking the $U(1)$ gauge symmetry down to $\mathbb{Z}_2$. The $U_{3rd\text{n.n.}}$ denotes bonds of length 2 connecting two sites and passing through a third site (such as the bond $1 \to 4$ in Fig. 1 in the main paper); instead, $\tilde{U}_{3rd\text{n.n.}}$ denotes bonds of length 2 which dont pass through any site.

FIG. 1. For the four $\mathbb{Z}_2$ spin liquids in the neighborhood of the $[0,0]$ state: The manner of variation of energy as the gauge breaking parameter (shown in red in Table I) is tuned on from zero to a small finite value, is shown. The parameters in black in Table I are fixed to their optimized values, and correspond to a suitably extended n^{th} n.n. gapless uniform RVB SL. The increase in energy upon opening a gap is apparent.

In the main paper, we have discussed the numerical results of a full Monte Carlo optimization for the four gapped $\mathbb{Z}_2$ SLs in the neighborhood of the $[0,0]$ state. In particular, we find that, by starting from arbitrary points in the variational space, one returns back to the gapless $[0,0]$ reference SL (suitably extended to n^{th} n.n.), with the value of the gauge breaking parameter going to zero. The energies of the n.n. $[0,0]$ SL and the extended 2nd and 3rd n.n. $[0,0]$ SLs correspond to the points B, A, C in Fig. 1 respectively, these energies are $E/J = -0.41216(1)$, $E/J = -0.41228(1)$, $E/J = -0.412308(1)$ respectively. Here, we show the *local* explicit manner of increase in energy of these extended gapless $[0,0]$ SLs upon addition of a *small* gap opening (gauge breaking) parameter (given in red in Table. I), like we did for the $\mathbb{Z}_2[0,\pi]\beta$ state in Fig. 2(c) of main paper. It should be noted that the case of $\mathbb{Z}_2[0,0]$A with $\zeta_{\text{R}} = 0$ is equivalent to the $\mathbb{Z}_2[0,0]$D case. Hence, we have the same plot for both SLs in Fig. 1.


$Z_2[0,0]A\ ;\ \zeta_R = 0$
$Z_2[0,0]A\ ;\ \zeta_R\{\Delta_2\}$
$Z_2[0,0]B$
$Z_2[0,0]C$
$Z_2[0,0]D$
-0.411
-0.4115
-0.412
B
A
C
Energy (per site)
-0.04
-0.02
0
0.02
0.04
Gauge breaking parameters