Spin Coherent Phenomena in Quantum Dots Driven by Magnetic Fields

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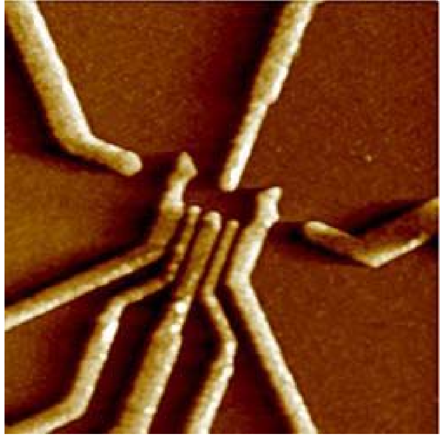
Toulouse, november 2010

Motivation and outline



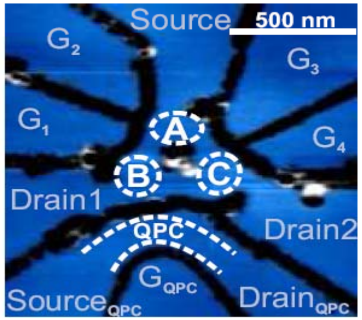
Semiconductor QD's: Small number of electrons, discrete levels: 2 level↑↓ system: q-bit.

Manipulation of q-bits in QD's:
driven coherent spin rotations

Electron Spin Resonance in Double Quantum Dots (ESR): Coherent Spin Rotations

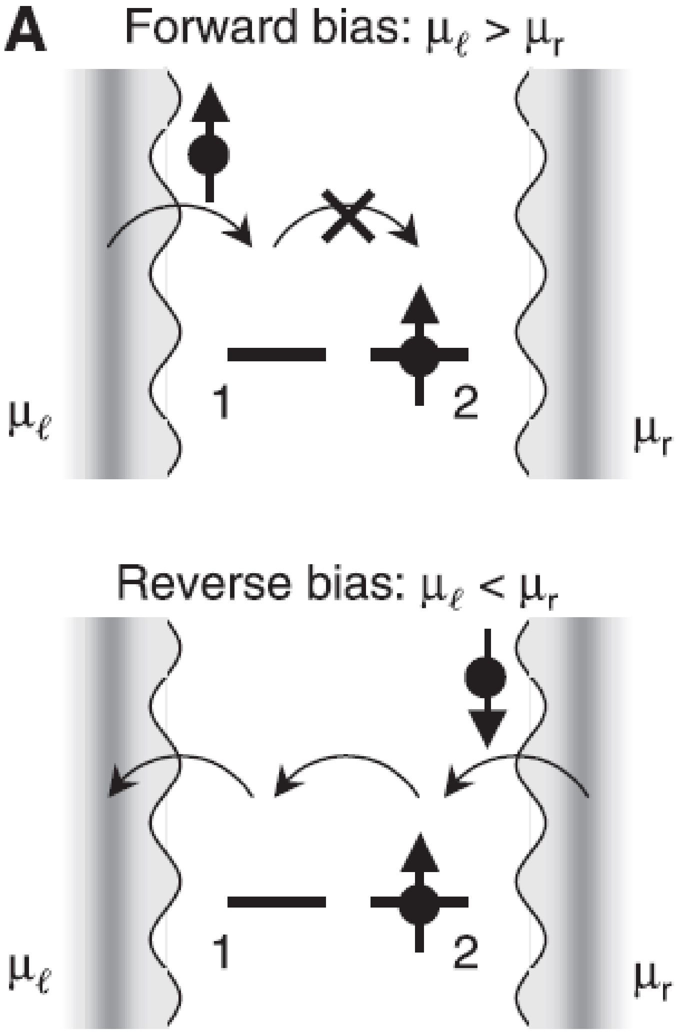
TQD's in magnetic fields: interplay between different coherent spin phenomena

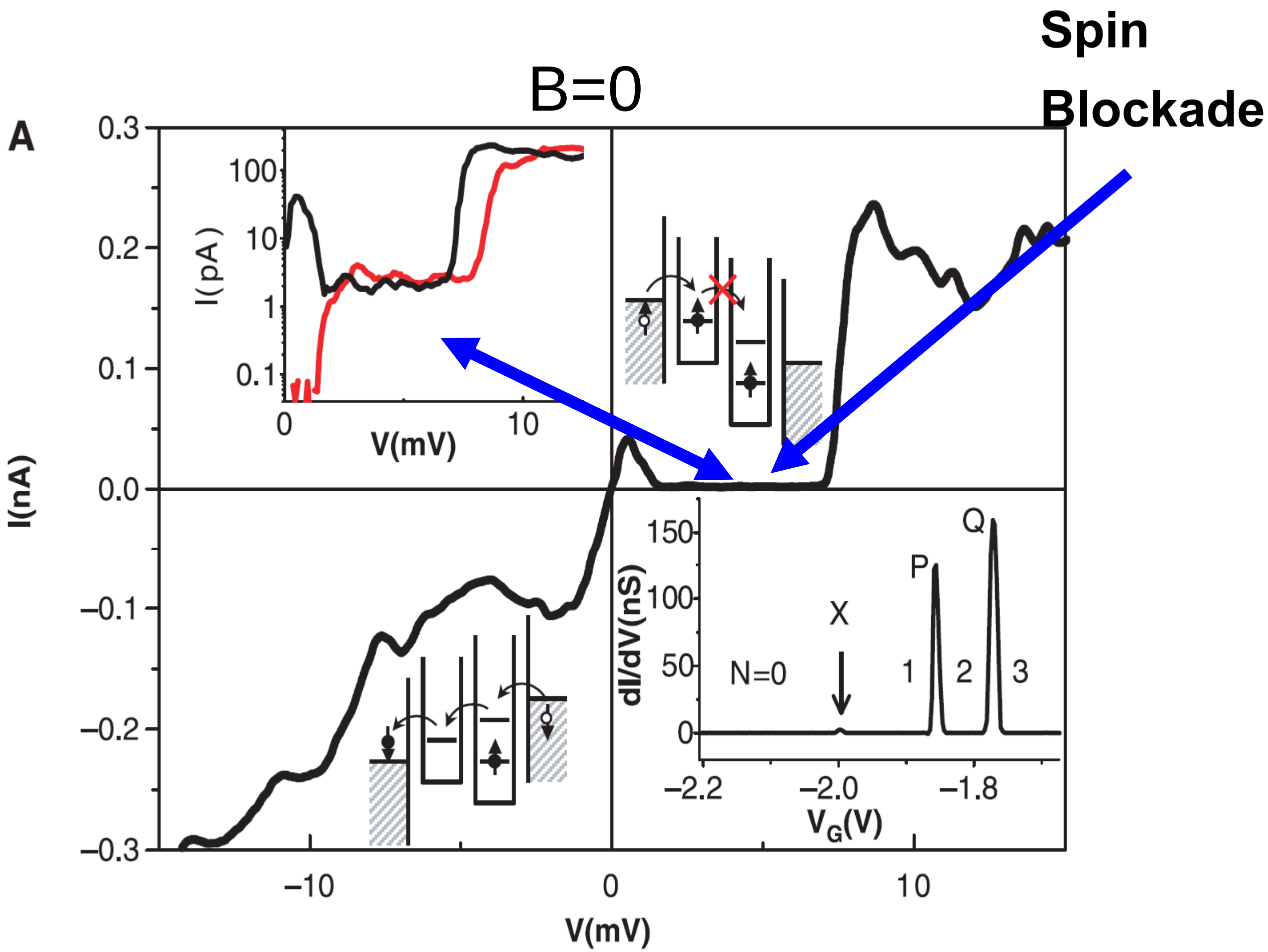
Triple Quantum Dots as step towards a network of QD's



Semiconductor QD's in magnetic fields: Spintronic devices: spin filters, spin inverters

Spin Blockade and Current Rectification by Pauli effect



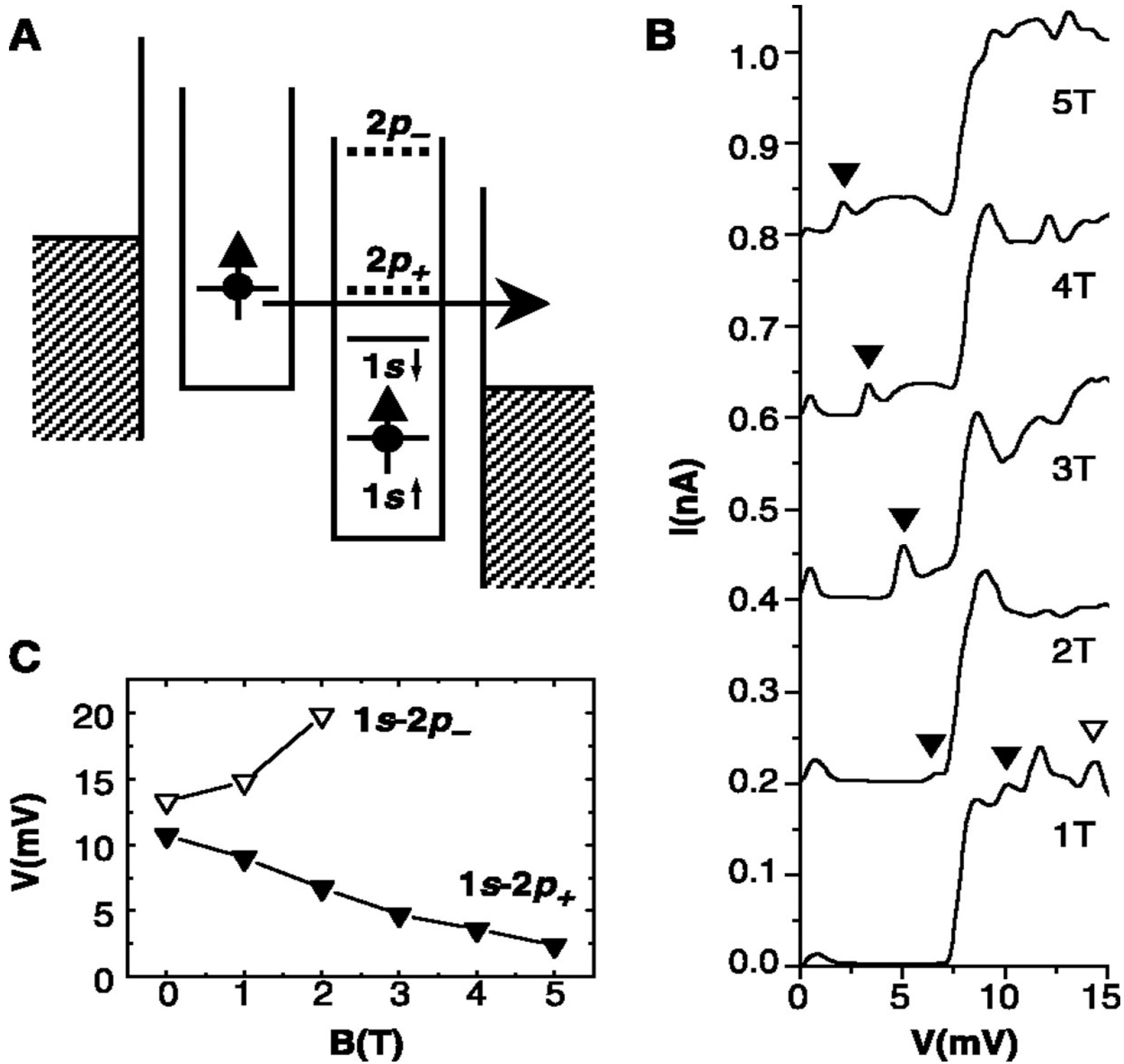


K. Ono et al., Science, 297 (02)

➢**Processes:** (0,1) → (1,1) → (0,2) → (0,1).

↓ Transport state

Removing Spin Blockade by an external magnetic field



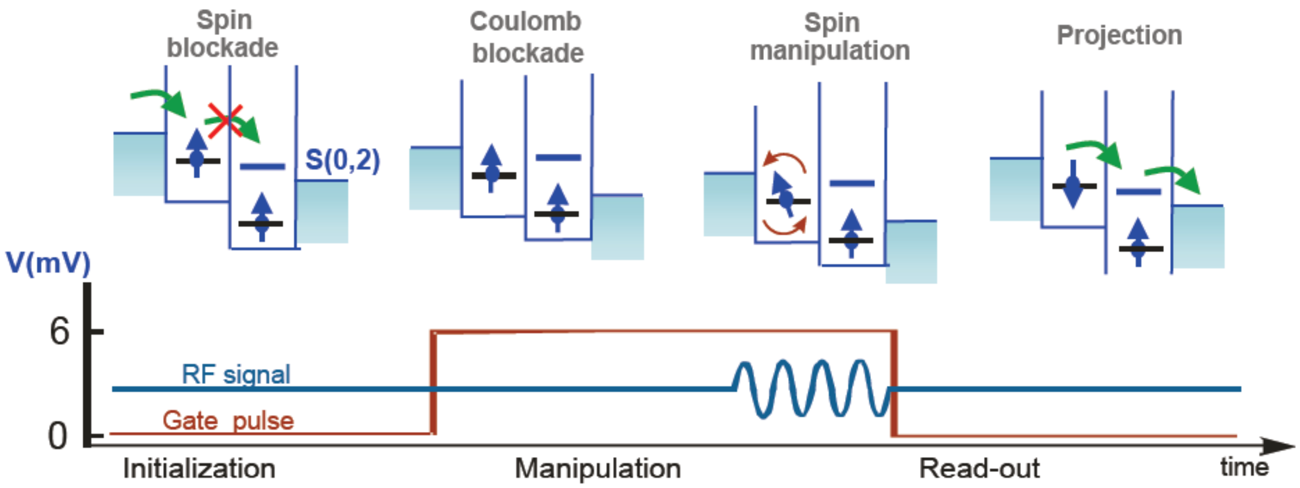
B parallel
to the current

K. Ono et al.,
Science, 297 (02)

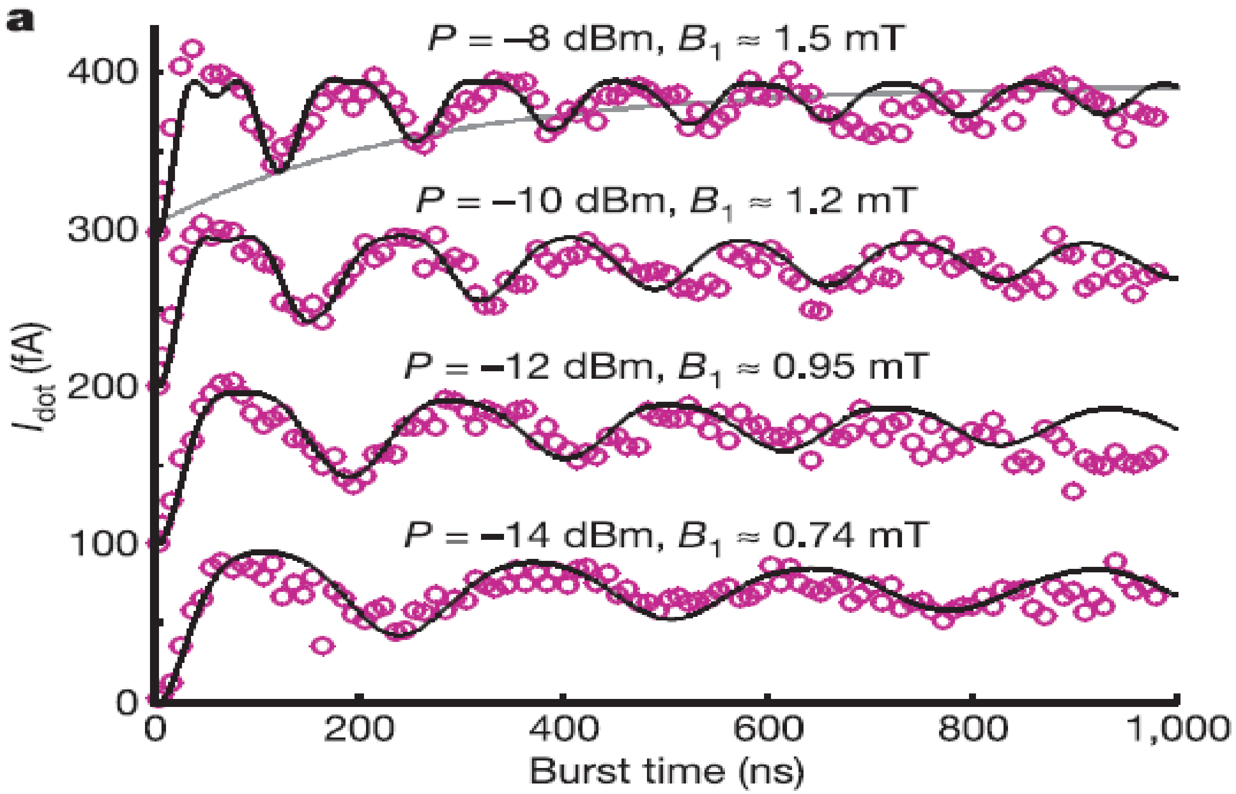
ESR in Quantum Dots

ESR: Driven coherent oscillations of an e spin in a DQD

Coherent manipulation: pulse scheme



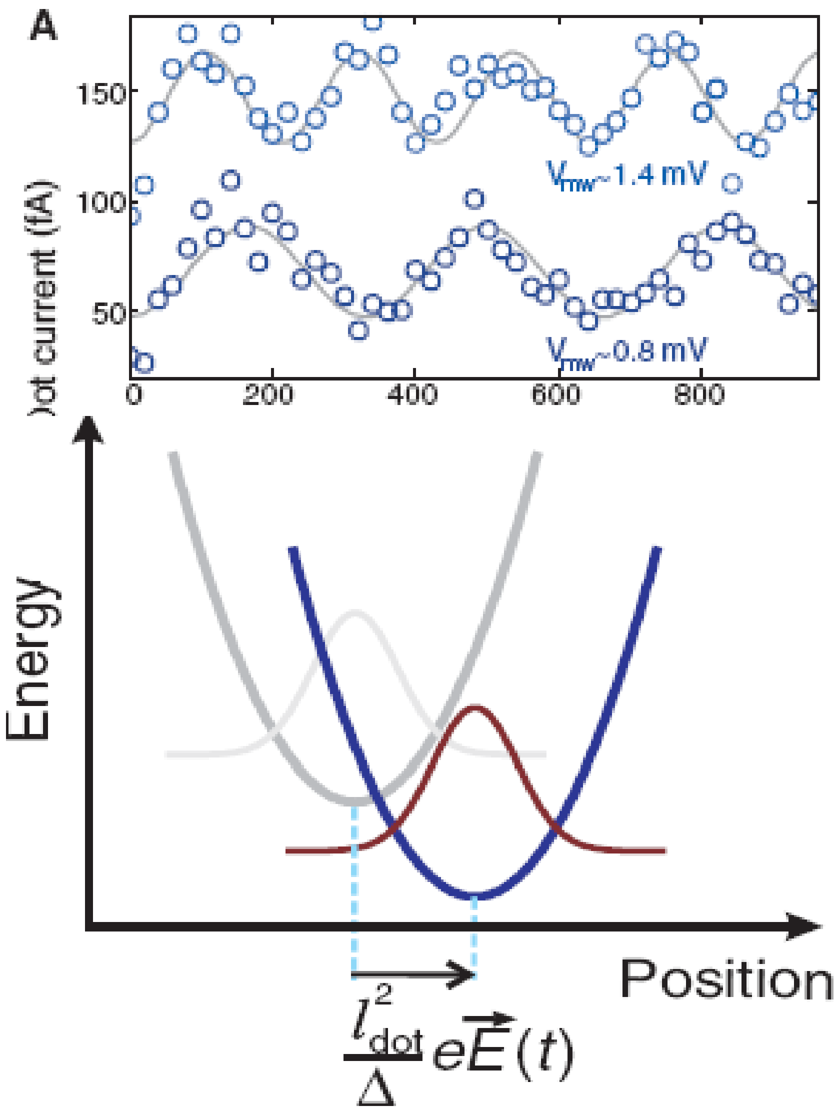
Koppens et al. Nature 2006



driven coherent oscillations of a single e spin in a DQD

Coherent Control of a Single Electron Spin with Electric Fields: Electron Dipole Spin Resonance **(EDSR)**

K. C. Nowack et al., Science 2007



ESR induced by AC electric fields in materials with a spin–orbit interaction:

Spin-Orbit

$$\mathrm{r}(x,y) = \mathbf{n} \otimes \mathbf{B}_{\text{ext}};\, n_x = \frac{2m^*}{\hbar}(-\alpha y - \beta x)$$

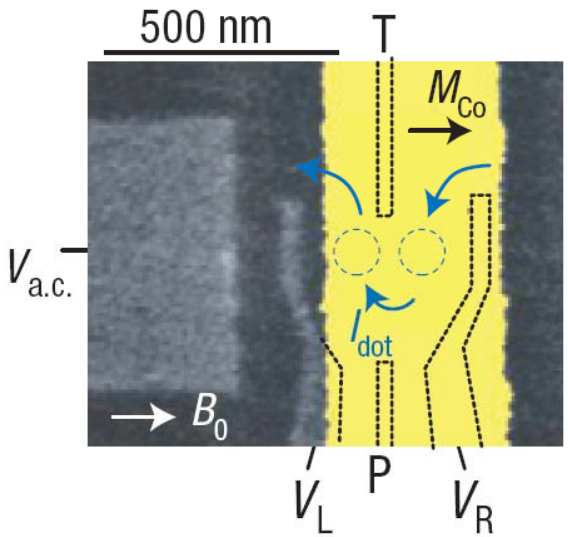
$$n_y = \frac{2m^*}{\hbar}(\alpha x + \beta y);\, n_z = 0$$

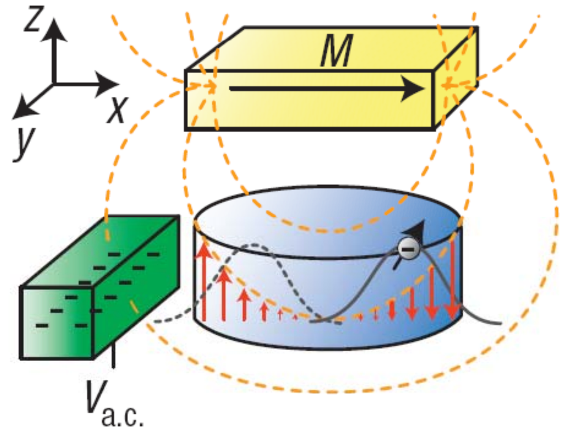
$$\mathbf{x}(t) = (e l_{\text{dot}}{}^2/\Delta)\mathbf{E}(t)$$

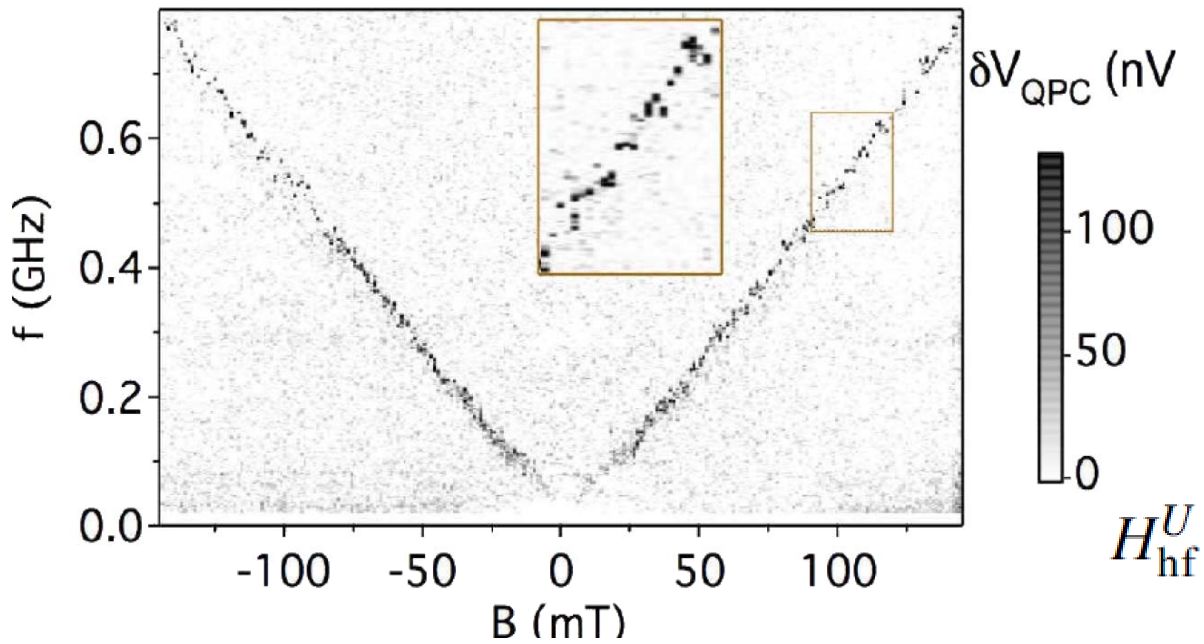
EDSR

Electrically driven single-electron spin resonance in a slanting Zeeman field

M. Pioro-Ladrière et al., Nature Physics 2008







Hyperfine-mediated gate-driven ESR

E. A. Laird et al., PRL 07

$$H_{\rm hf}^{U} = A\Sigma_j \delta(\mathbf{r} + \mathbf{R}(t) - \mathbf{r}_j)(\mathbf{I}_j \cdot \mathbf{S})$$

$$\mathbf{R}(t) = -e\tilde{\mathbf{E}}(t)/m\omega_0^2.$$

Coherent control of two spins

ac B_1 conventional magnetic resonance

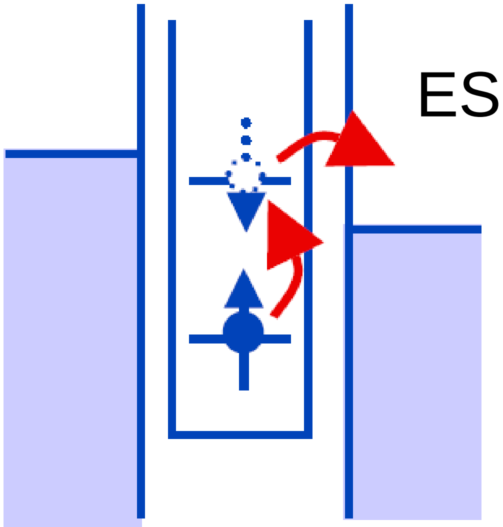
ac E_1 electrical dipole spin resonance, mediated by

(1) spin-orbit coupling

(2) gradient magnetic field

(3) hyperfine coupling to nuclear spins

ESR:Theoretical models:



1 QD: **Engel et al., PRL, 2001**

Rudner et al. PRL, 2007

Danon et al., PRL, 2008

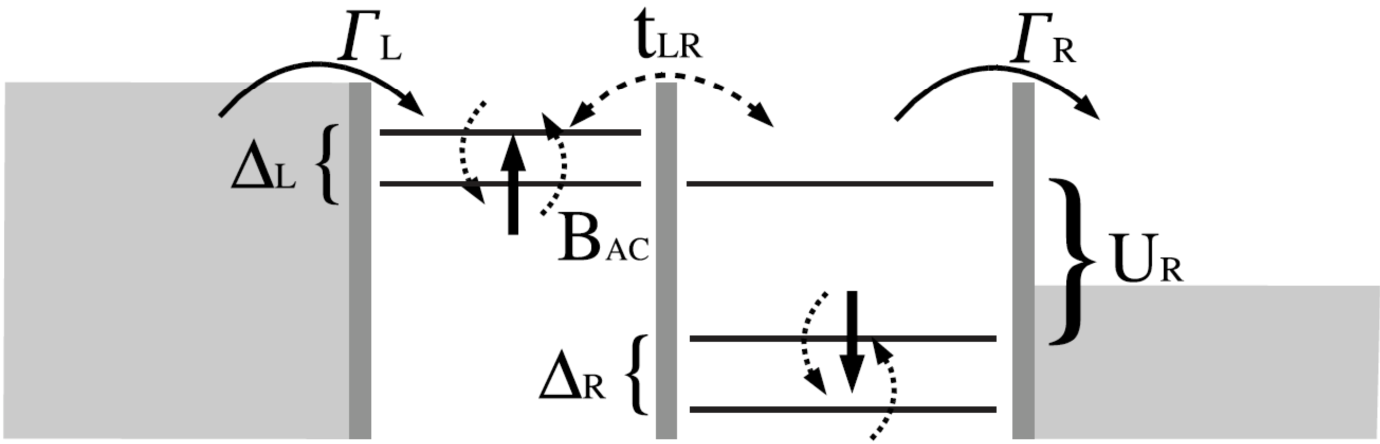
2 QDs:

R. Sánchez et al. PRB, 2008

J. Danon et al., PRL 09

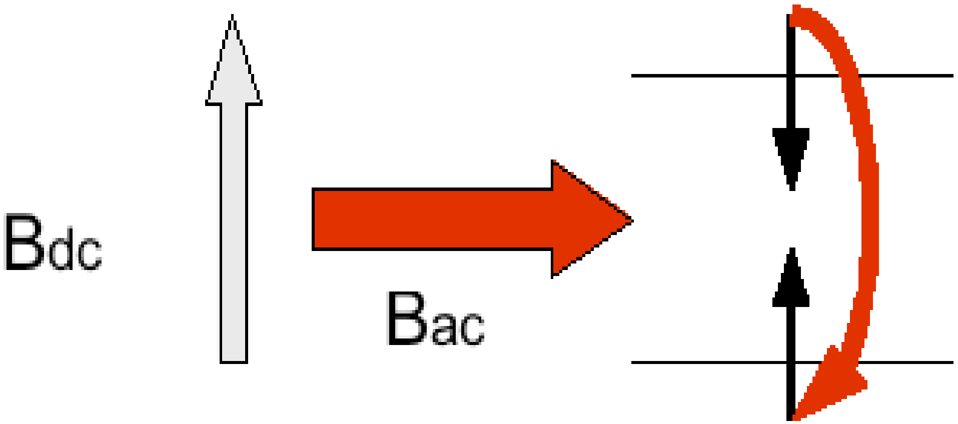
3 QDs: **M. Busl et al. PRB (RC) 2010; PRB 2010**

Electron Spin Resonance (ESR) in DQDs



$$\hat{H}_B(t) = \sum_{i=1} [\Delta_i S_{zi} + B_{ac}(\cos(\omega t) S_{xi} + \sin(\omega t) S_{yi})]$$

2 electrons:
Bac only acts on triplet states



Hamiltonian

$$\hat{H}_0(t) = \hat{H}_{QD} + \hat{H}_{tun} + \hat{H}_B(t) + \hat{H}_{QD-leads} + \hat{H}_{leads}$$

Master equation: Born-Markov Approximation

Coherent dynamics

$$\begin{aligned}\dot{\rho}_{ln}(t) = & -i \quad \langle l|[\hat{H}_{\mathrm{QD}} + \hat{H}_{\mathrm{t}} + \hat{H}_{\mathrm{B}}(t), \rho]|n\rangle \\ & + \sum_{k \neq n} (\Gamma_{nk}\rho_{kk} - \Gamma_{kn}\rho_{nn})\delta_{ln} \\ & - \quad \Lambda_{ln}\rho_{ln}(1 - \delta_{ln})\end{aligned}$$

Transition rates and decoherences

the state which contributes to the current is : $|0, \uparrow\downarrow\rangle$

2 electrons: Singlets and Triplets

Coherently coupled by inter-dot tunneling

$$\left|S_0\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\uparrow,\downarrow\right\rangle - \left|\downarrow,\uparrow\right\rangle\right)$$

$$\left|S_g\right\rangle = \left|0,\uparrow\downarrow\right\rangle$$ **Transport state**

$\left|S_T = 0\right\rangle$

H_{Tunnel}

conserves total spin

$$\left|T_{+1}\right\rangle = \left|\uparrow,\uparrow\right\rangle$$

$$\left|T_0\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\uparrow,\downarrow\right\rangle + \left|\downarrow,\uparrow\right\rangle\right)$$

$$\left|T_{-1}\right\rangle = \left|\downarrow,\downarrow\right\rangle$$

$\left|S_T = 1\right\rangle$

B_{ac}

acts on the triplet subspace

$B^{Left} = B^{Right}$ → No S-T mixing

if $B^{Left} \neq B^{Righ}$ →

$$\left|\Psi\right\rangle \propto \left|S_T = 1\right\rangle + \delta\left|S_T = 0\right\rangle$$

$$\delta = -\left|g\right|\mu_B\left(B^{Right} - B^{Left}\right)$$

Dependence of the time averaged current on the Zeeman inhomogeneity

R. Sánchez et al., PHYSICAL REVIEW B **2008**

$$\frac{\Delta_L}{\Delta_R} \neq 1$$

$$\frac{1}{\sqrt{2}}\left(|\uparrow,\downarrow\rangle + |\downarrow,\uparrow\rangle\right)$$

mixes with singlet state

$$|S_0\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow,\downarrow\rangle - |\downarrow,\uparrow\rangle\right)$$

$$|S_g\rangle = |0,\uparrow\downarrow\rangle$$ Transport state

$$B_{ac} \neq 0$$

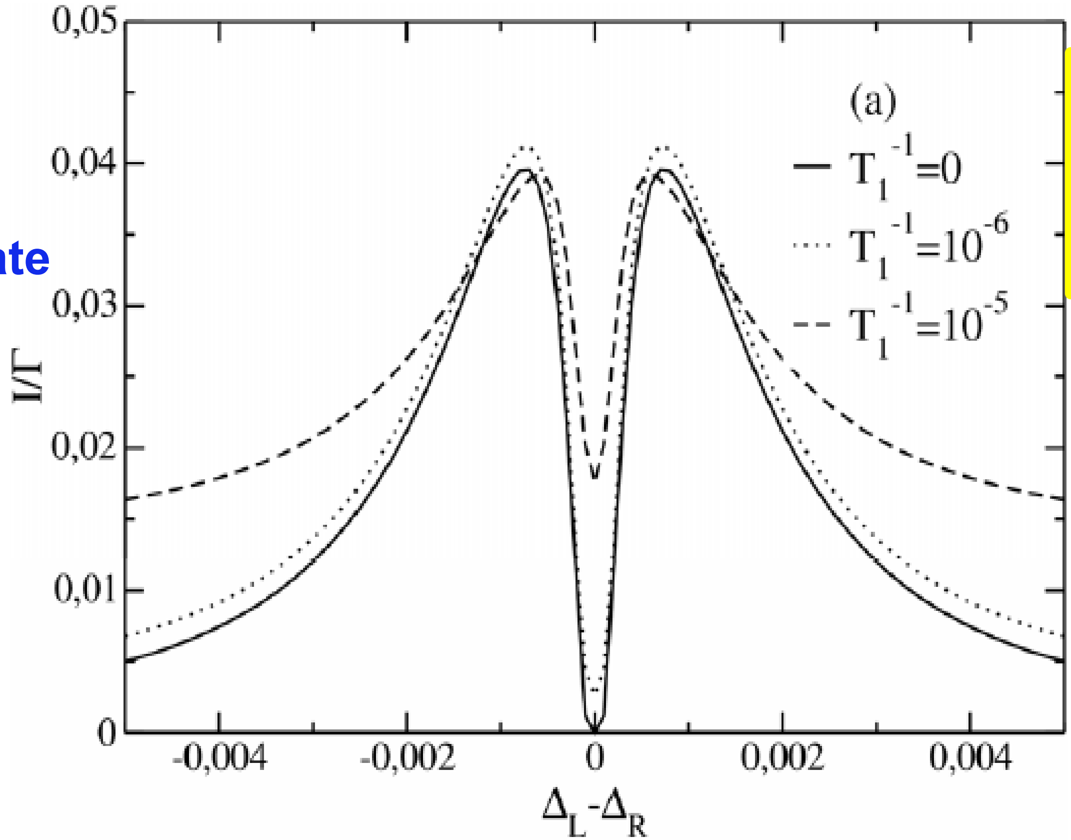
$$|\downarrow,\downarrow\rangle$$

$$|\uparrow,\uparrow\rangle \rightarrow \frac{1}{\sqrt{2}}\left(|\uparrow,\downarrow\rangle + |\downarrow,\uparrow\rangle\right)$$

↓

$$I \neq 0$$

$$\Omega_{AC} = \Omega_T \qquad \Delta_L = \omega$$



$$B_{ac} \neq 0$$

removes SB

↓

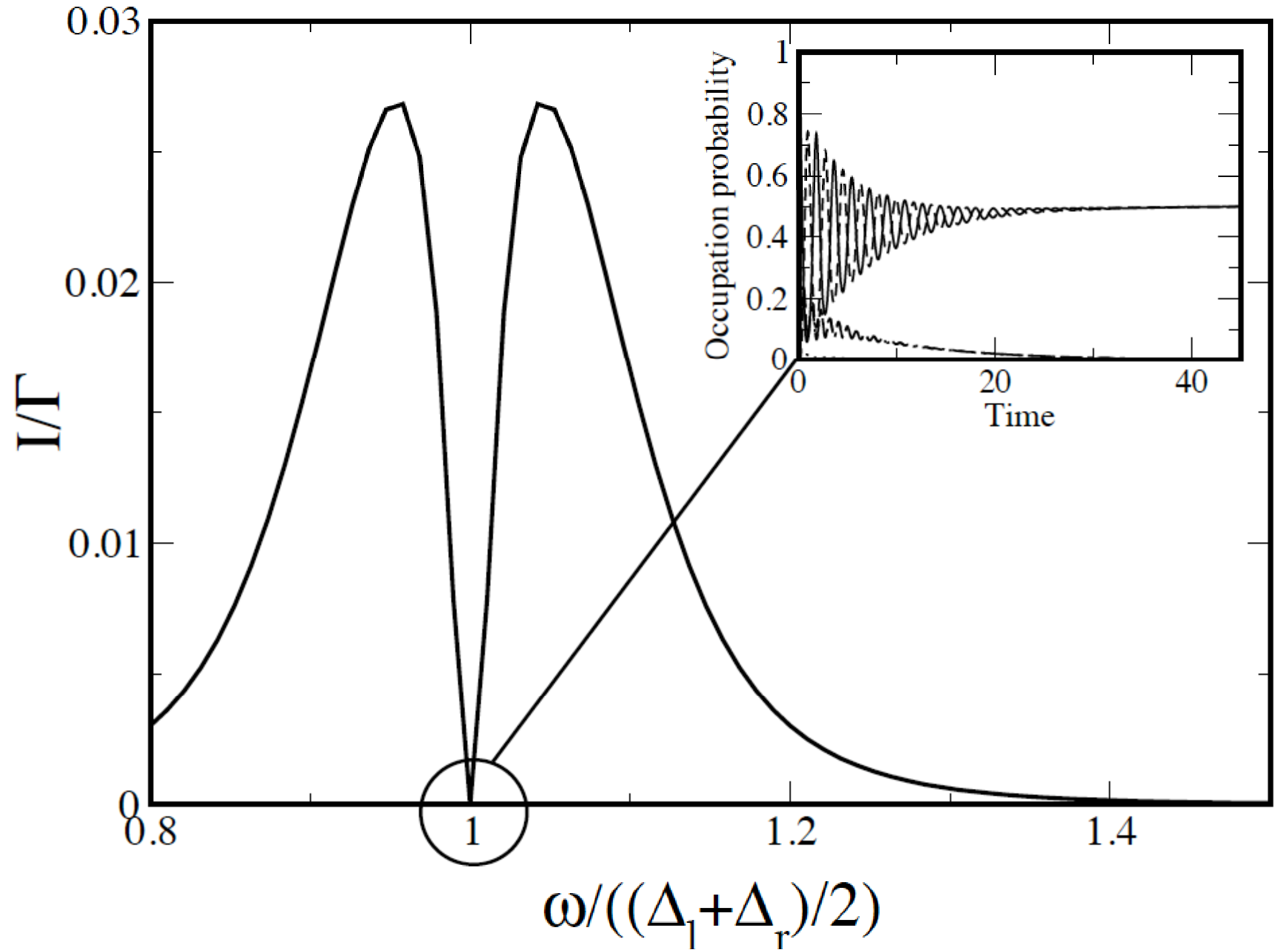
Current drops to zero by Spin Blockade

$$\Delta_{zL} \neq \Delta_{zR}$$

$$B_{AC} \neq 0$$

BAC removes SB, but also:

B_{AC} **induces spin blockade at** $\omega = \frac{\Delta_L + \Delta_R}{2}$



a coherent superposition of $|\uparrow,\uparrow\rangle$ **and** $|\downarrow,\downarrow\rangle$ **is formed:**

$$\frac{|\uparrow,\uparrow\rangle - |\downarrow,\downarrow\rangle}{\sqrt{2}}$$

$$\Delta_{zL} \neq \Delta_{zR}$$

$$\Delta_{zR} = 0.9\Delta_{zL}$$

Hamiltonian

$$\hat{H}(t) = \Delta_{zL}\hat{S}_{zL} + \Delta_{zR}\hat{S}_{zR} + B_{ac}\left(cos(\omega t)(\hat{S}_{xL} + \hat{S}_{xR}) + sin(\omega t)(\hat{S}_{yL} + \hat{S}_{yR})\right)$$

Unitary Transformation $\hat{U}(t) = e^{-i\sum_j \omega_j \hat{S}_z^j t}$

$$\hat{H}' = \Delta_{zL}\hat{S}_{zL} + \Delta_{zR}\hat{S}_{zR} - \hbar\omega(\hat{S}_{zL} + \hat{S}_{zR}) + B_{ac}(\hat{S}_{xL} + \hat{S}_{xR})$$

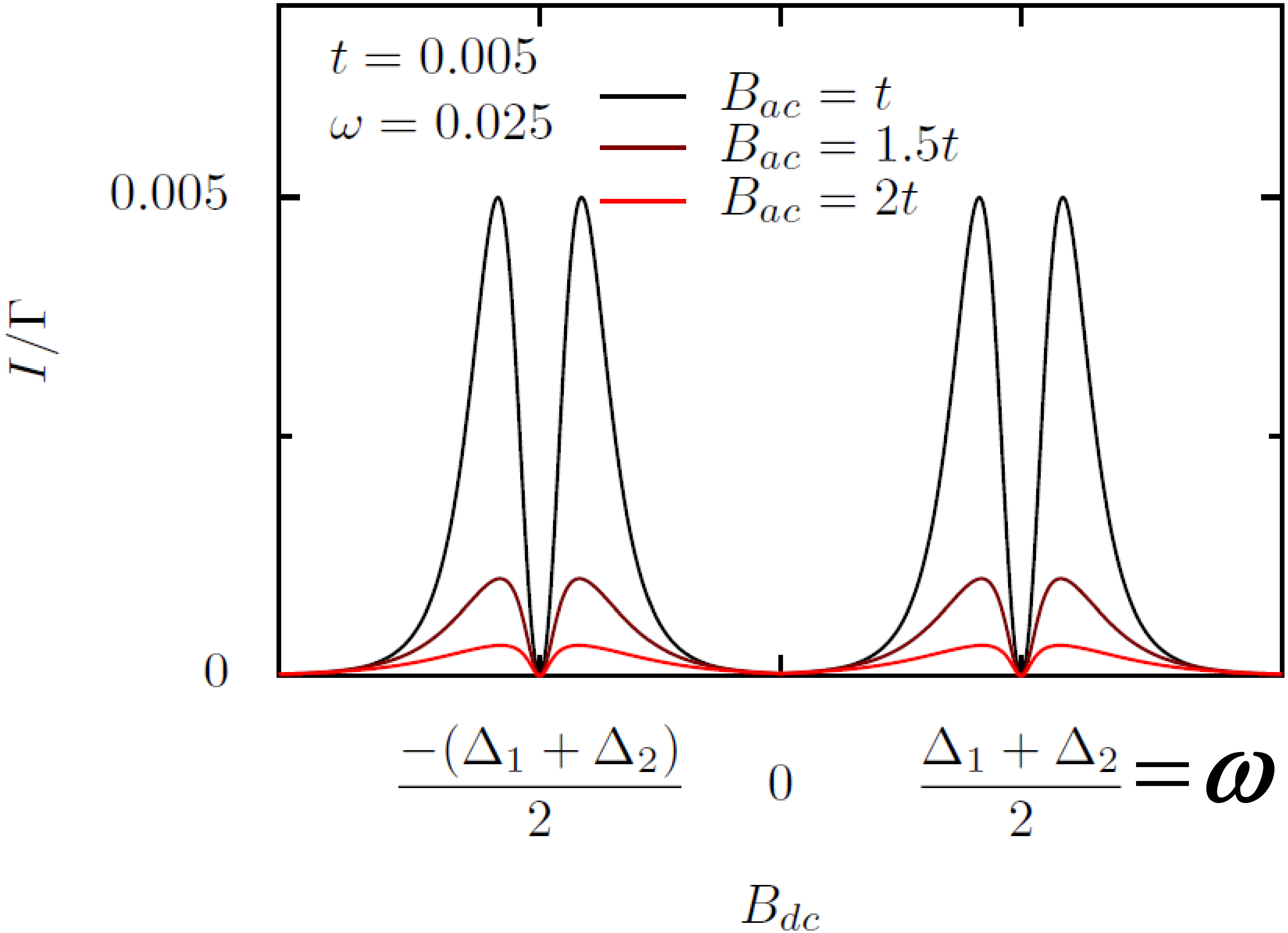
$$\hat{H}'\left(|\uparrow,\uparrow\rangle - |\downarrow,\downarrow\rangle\right) = 0 \quad \text{for} \quad \omega = \frac{\Delta_L + \Delta_R}{2}$$

back transformed state: $|\Psi(t)\rangle = e^{-i\frac{(\Delta_{zL}+\Delta_{zR})t}{2}}|\uparrow,\uparrow\rangle - e^{i\frac{(\Delta_{zL}+\Delta_{zR})t}{2}}|\downarrow,\downarrow\rangle$

This state verifies the time-dependent Schrodinger equation and:

$H(t)|\psi(t)\rangle$ =0

$$\hat{H}(t) = \sum_{i=1} B_{ac}(\cos(\omega t)S_{xi} + \sin(\omega t)S_{yi})$$

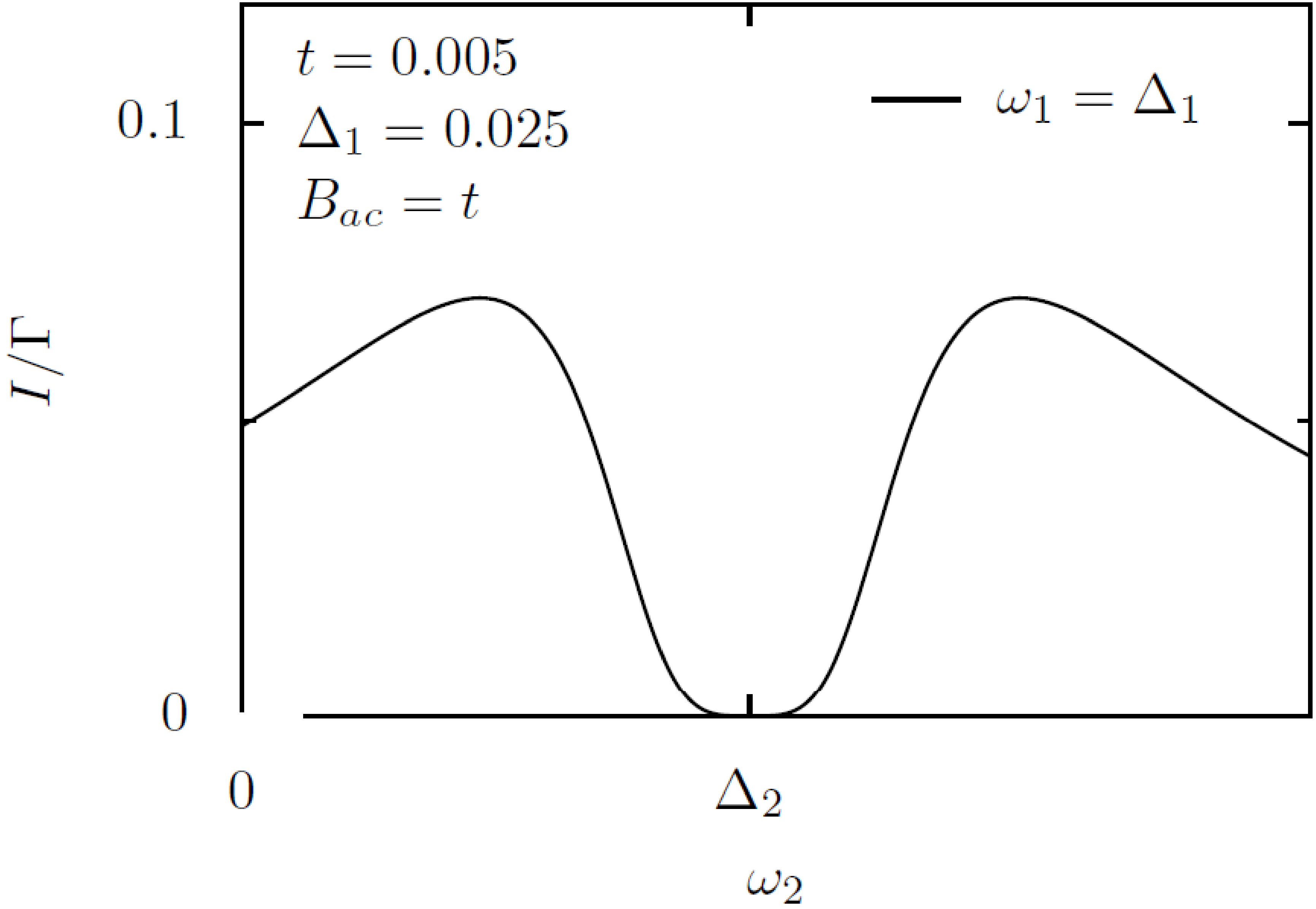


$t = 0.005$
$\omega = 0.025$
$B_{ac} = t$
$B_{ac} = 1.5t$
$B_{ac} = 2t$
0.005
0
I/Γ
$\frac{-(\Delta_1 + \Delta_2)}{2}$
0
$\frac{\Delta_1 + \Delta_2}{2} = \omega$
B_{dc}

Bichromatic magnetic field

$$\Delta_1 = \omega_1$$
$$\Delta_2 = \omega_2$$

At resonace conditions Spin Blockade is induced:

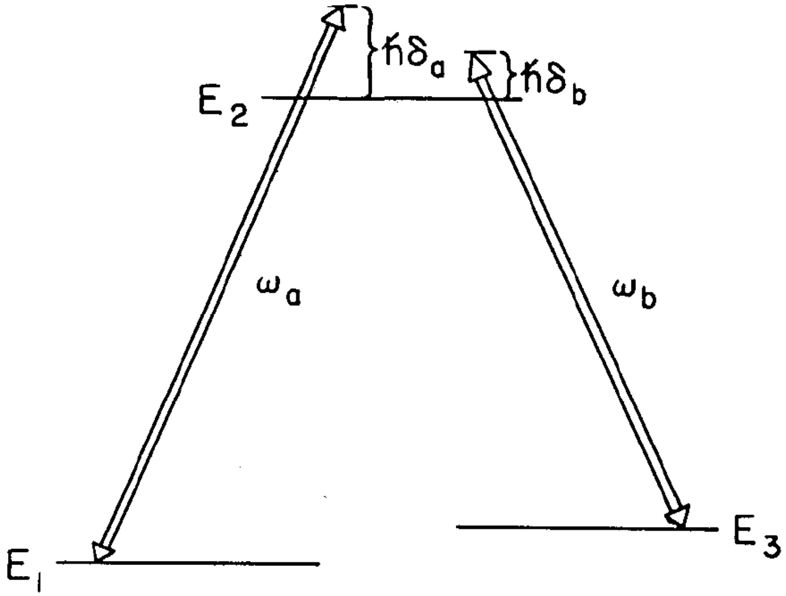


Coherent Trapping of Atomic Populations

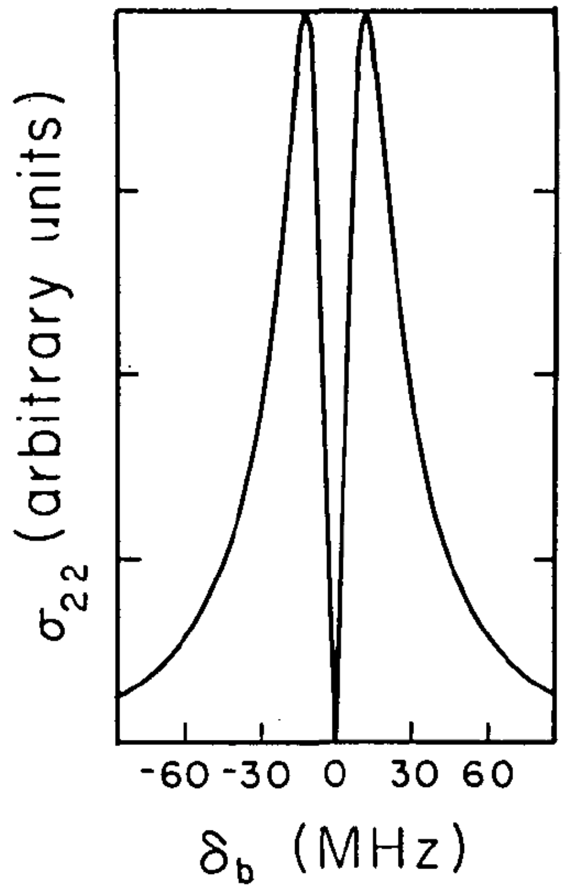
H. R. Gray et al., Optics Letters, 1978

E. Arimondo et al., Nuevo Cimento Lett 76
G. Alzetta et al., Nuovo Cimento 76

Superposition of two splitted ground states which is decoupled from the light



Dark state



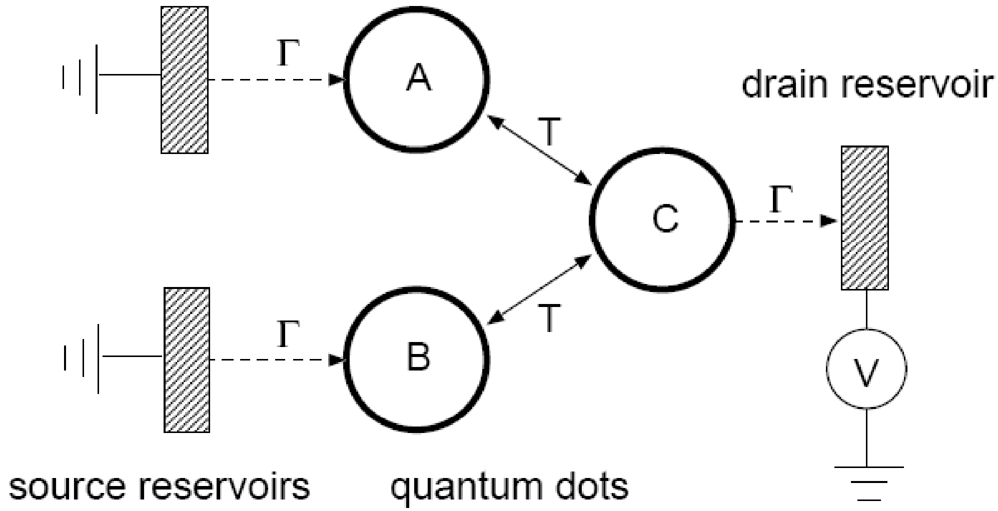
$$\lim_{t\to\infty} \psi(r,t) = \cos\theta \exp(-i\omega_1 t)\psi_1(r) - \sin\theta \exp(-i\omega_3 t)\psi_3(r)$$

CPT in QD's: Brandes et al., PRL 00

Current Switch by Coherent Trapping of Electrons in Quantum Dots

All-electronic coherent population trapping in quantum dots

B. Michaelis, C. Emary and C. Beenakker, EPL 06



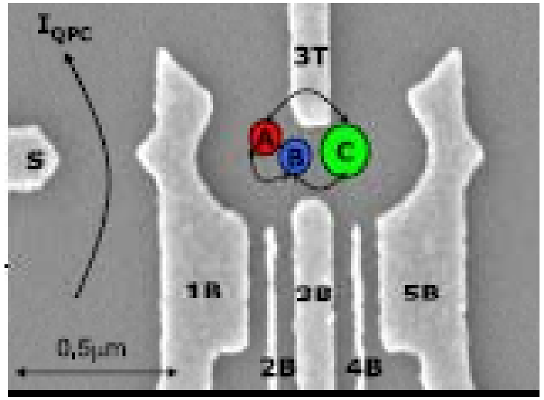
$$|\Phi_-\rangle = \frac{\left(|A\rangle - |B\rangle\right)}{\sqrt{2}}$$

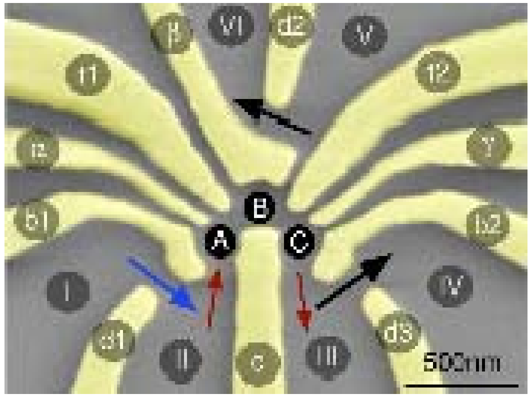
Dark State

Destructive interference of the two reversible transitions traps an electron in a coherent superposition of the states on dots A and B.

Experiments in TQDs

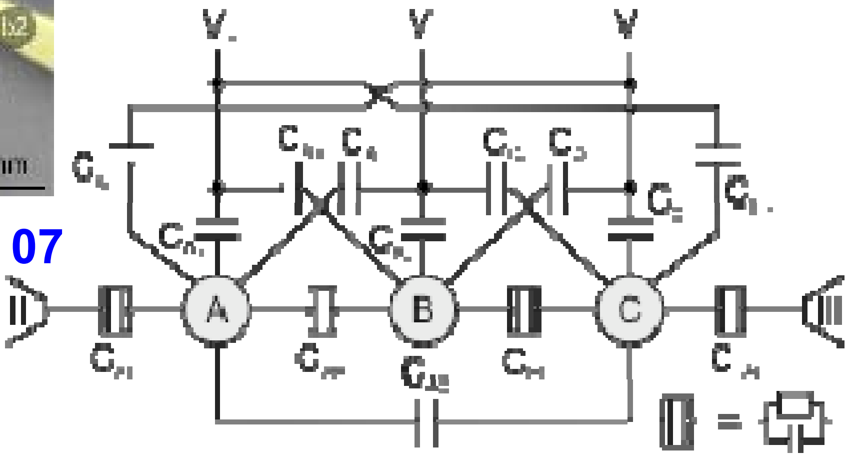
Gaudreau et al., PRL 2006





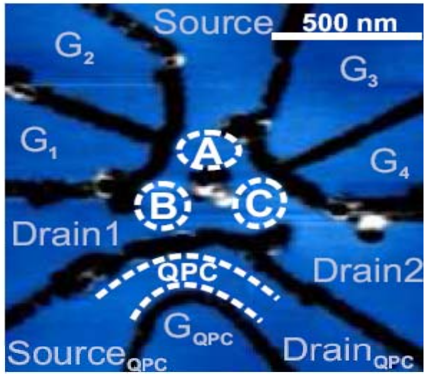
Schröer et al., PRB 07

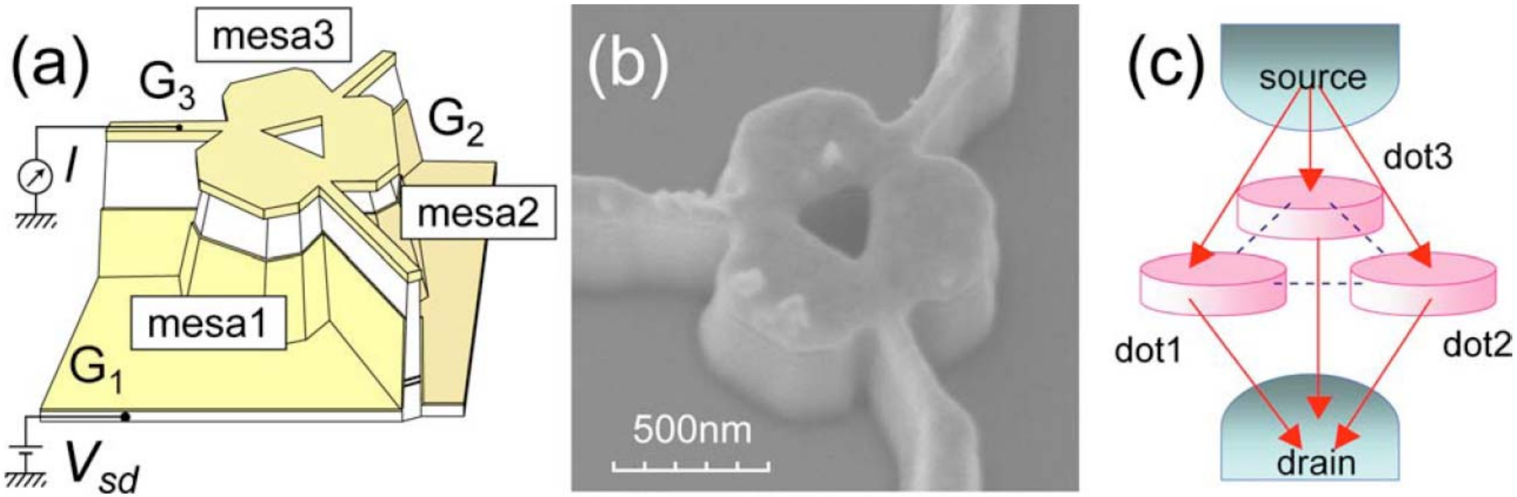
TQD's in series



Equivalent circuit

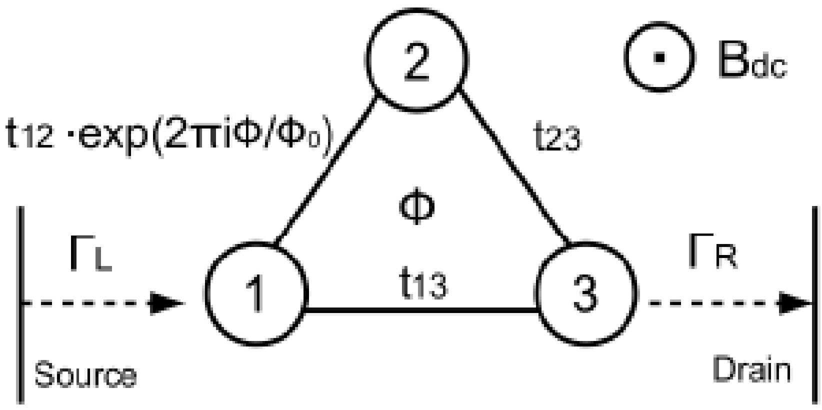
Rogge et al., PRB 08





Laterally coupled triple vertical quantum dots in triangular arrangement, S. Amaha et al., APL 09

TQD triangular configuration: 1 electron



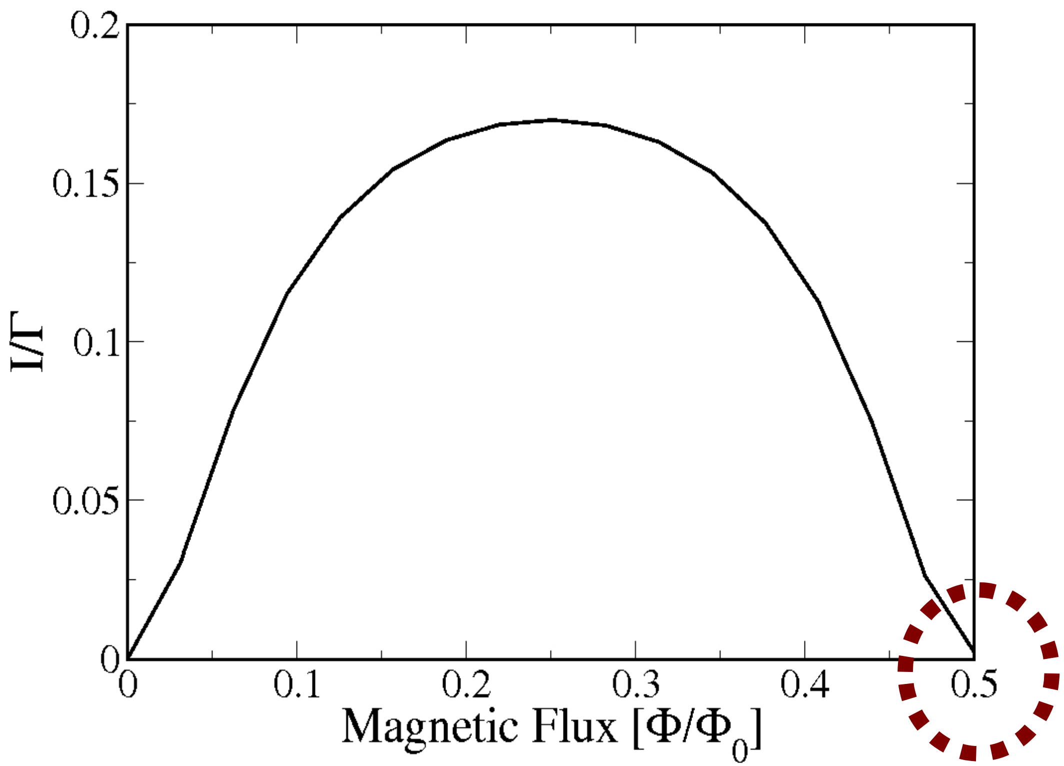
Dark states:

$$\frac{1}{\sqrt{2}}\left(\left|0,\uparrow,0\right\rangle - \left|\uparrow,0,0\right\rangle\right)$$

$$\frac{1}{\sqrt{2}}\left(\left|0,\downarrow,0\right\rangle - \left|\downarrow,0,0\right\rangle\right)$$

do not contribute to the current due to destructive interference of the tunneling to dot 3

Aharonov-Bohm current oscillations



$$\Phi = \boldsymbol{n}\frac{\Phi_0}{2}$$

Dark states remain and I=0

$$\Phi \neq \boldsymbol{n}\frac{\Phi_0}{2}$$

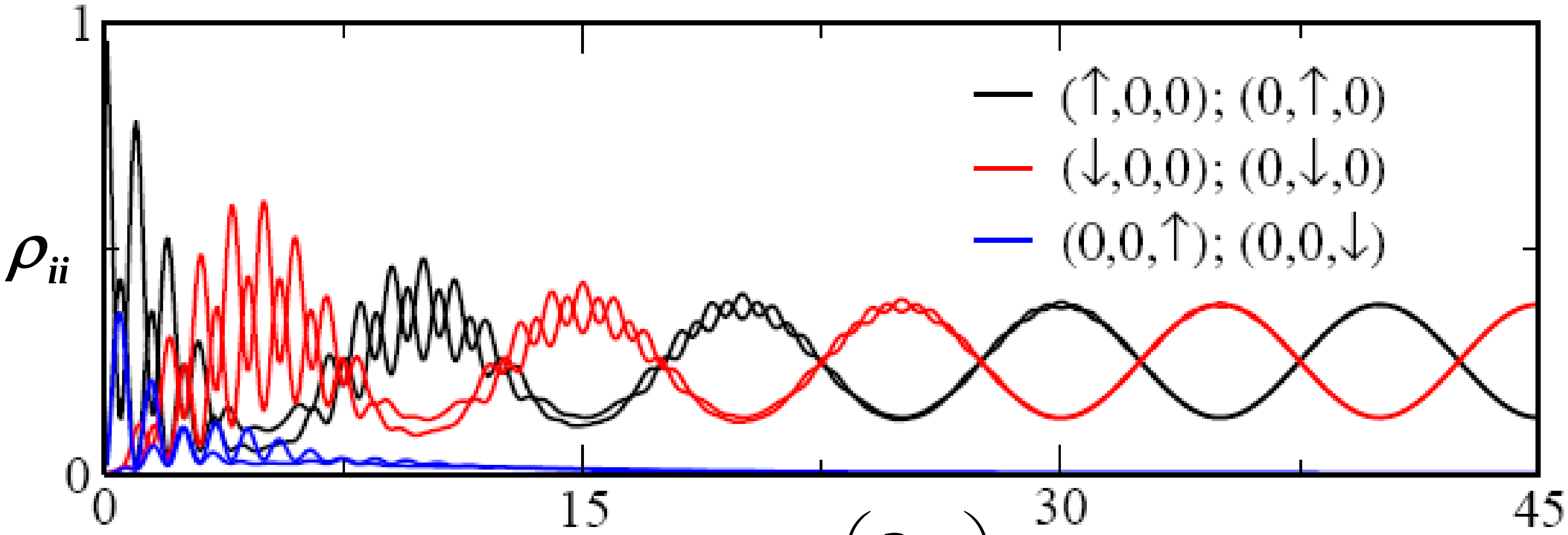
Dark states are destroyed

$$\Phi = \frac{\Phi_0}{2}$$

Triangular TQD with 1 electron: ESR $\quad B_{ac} \neq 0$

$$\Delta_1 = \Delta_2 = \Delta_3 = \omega_{ac}$$

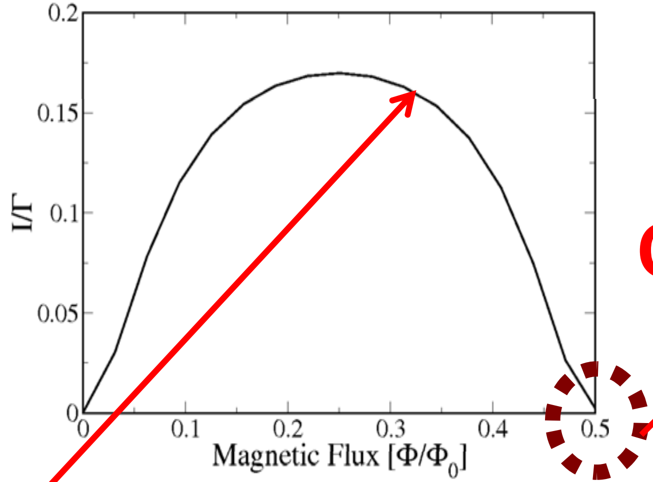
$$\frac{1}{\sqrt{2}}\left(|0,\uparrow,0\rangle - |\uparrow,0,0\rangle\right) \Longleftrightarrow \frac{1}{\sqrt{2}}\left(|0,\downarrow,0\rangle - |\downarrow,0,0\rangle\right)$$



$I = 0$

$\Phi = \frac{\Phi_0}{2}$

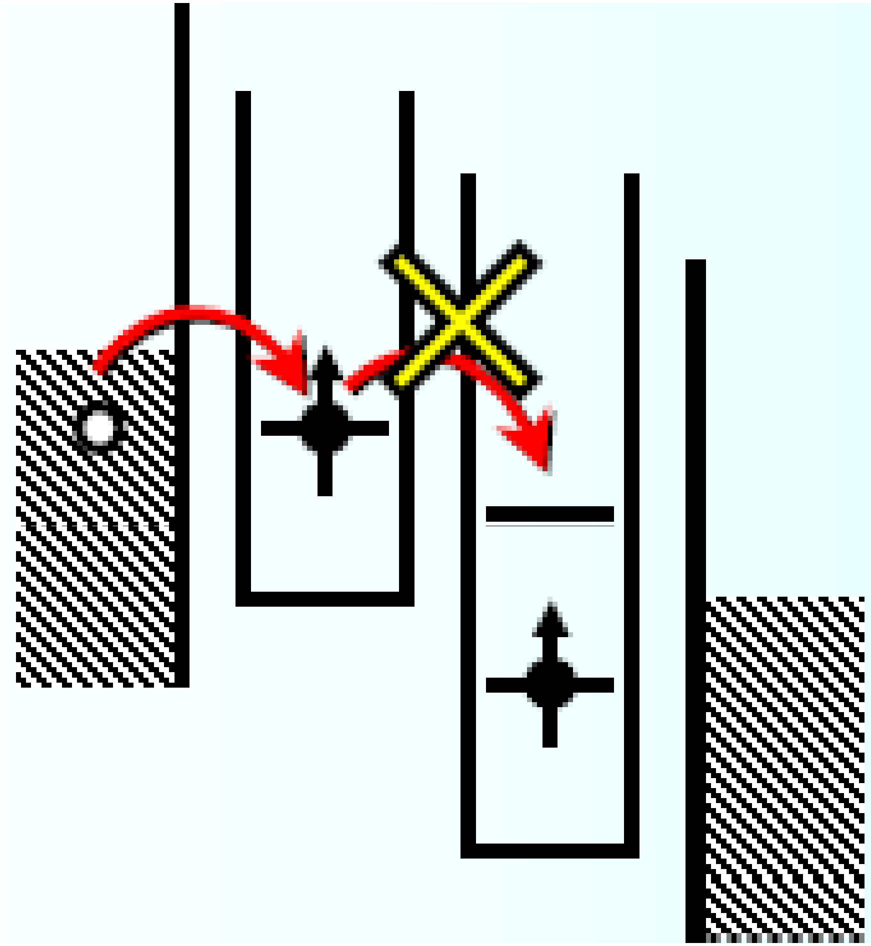
Time $\left(\frac{2\pi}{\Omega_t}\right)$

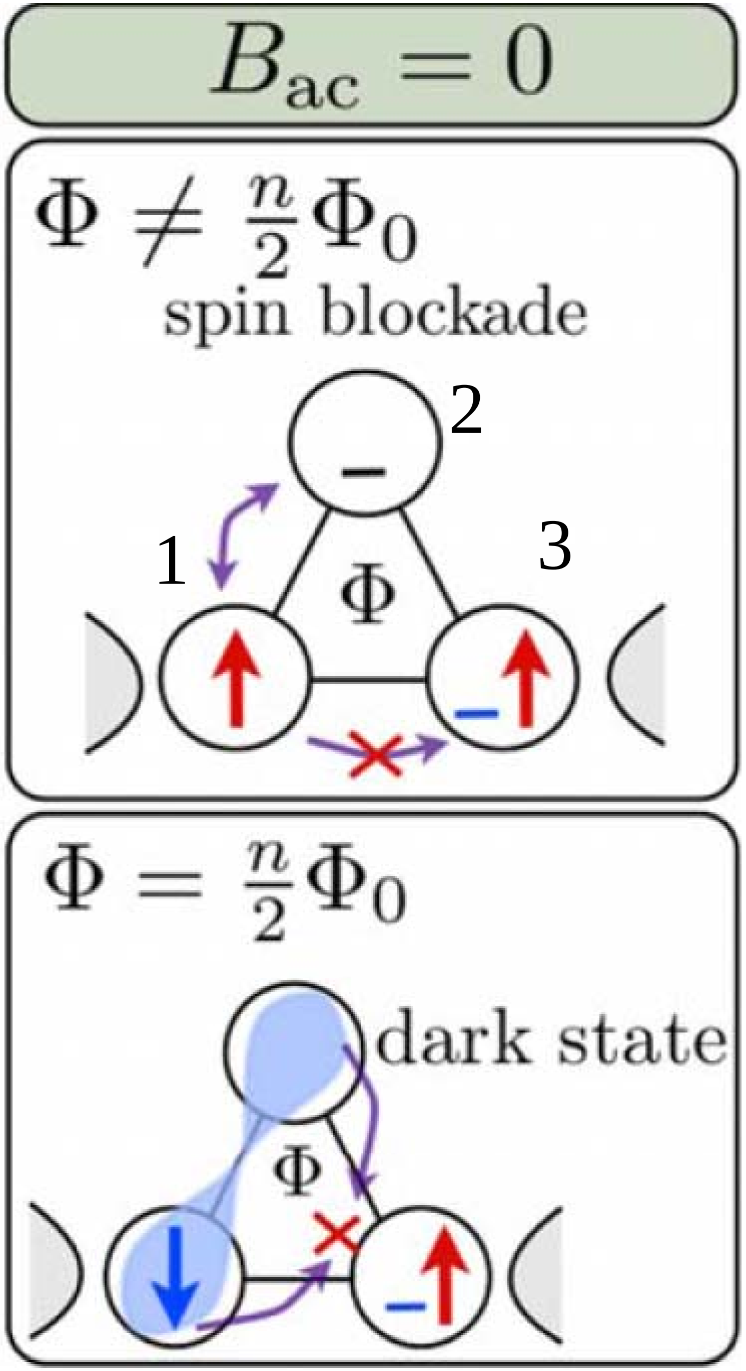


Coherent rotations of 1 single electron

$\Phi \neq n\frac{\Phi_0}{2}$ ⇒ **Dark States are destroyed and** $I \neq 0$

2 electrons: Spin Blockade





TQD with 2 electrons $\Delta_1 = \Delta_2 \neq \Delta_3$

Transport state: $|S\rangle = |0,0,\uparrow\downarrow\rangle$

$$I(t) \propto \Gamma \rho_{S,S}(t)$$

Spin Blockade → $I = 0$

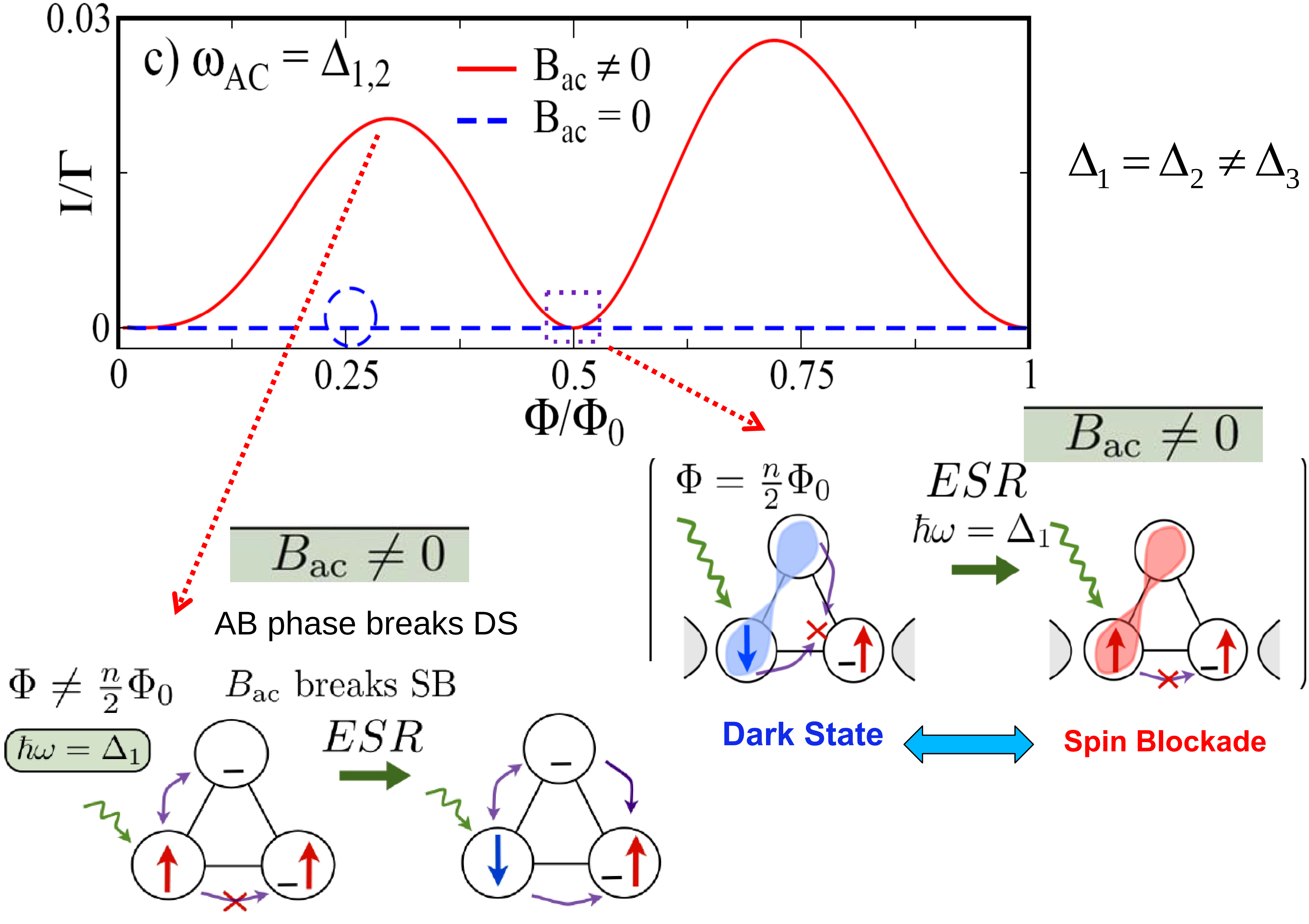
$$|s,0,s\rangle \Leftrightarrow |0,s,s\rangle$$

Dark State and SB

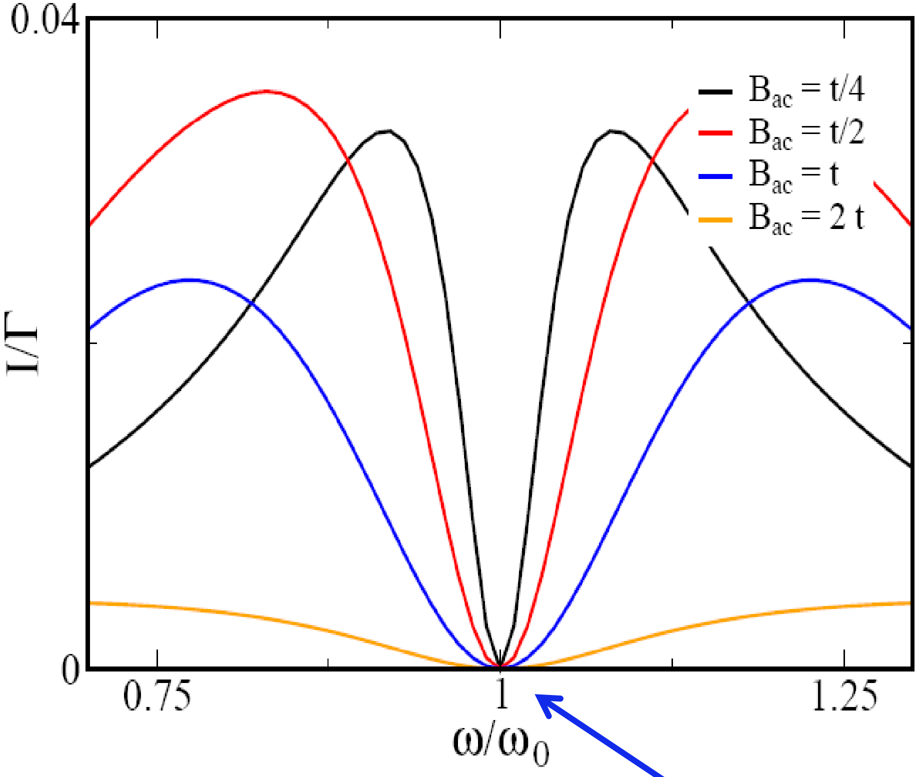
$$|\Psi_{ss'}\rangle = \frac{1}{\sqrt{2}}\left(|0,s,s'\rangle - |s,0,s'\rangle\right)$$

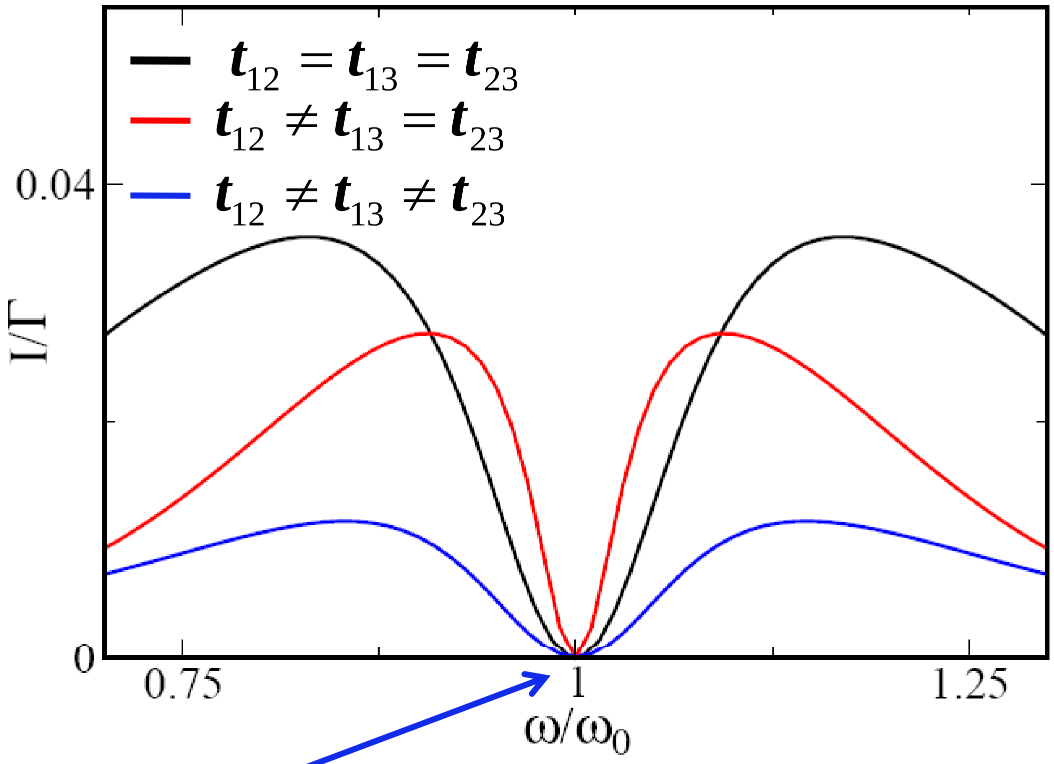
$s \neq s'$

$$\langle \Psi_{s,s'} | H_{tun} | S \rangle = 0 \longrightarrow I = 0$$



c) $\omega_{AC} = \Delta_{1,2}$
$B_{ac} \neq 0$
$B_{ac} = 0$
0.03
0
I/Γ
0
0.25
0.5
0.75
1
Φ/Φ_0
$\Delta_1 = \Delta_2 \neq \Delta_3$
$B_{ac} \neq 0$
$\Phi = \frac{n}{2}\Phi_0$
ESR
$\hbar\omega = \Delta_1$
Dark State
Spin Blockade
$B_{ac} \neq 0$
AB phase breaks DS
$\Phi \neq \frac{n}{2}\Phi_0$
B_{ac} breaks SB
$\hbar\omega = \Delta_1$
ESR





$$\Phi = 0.25\Phi_0$$
$$\Delta_1 = \Delta_2 \neq \Delta_3$$
$$\Delta_3 = 0.75\Delta_1$$

Bac induces Spin Blockade

Eigenstate at: $$\omega_0 = \frac{\Delta_1 + \Delta_3}{2}$$

$$|\Psi\rangle = \frac{1}{2}\left(\boldsymbol{i}|\uparrow,0,\uparrow\rangle - \boldsymbol{i}|\downarrow,0,\downarrow\rangle - |0,\uparrow,\uparrow\rangle + |0,\downarrow,\downarrow\rangle\right)$$

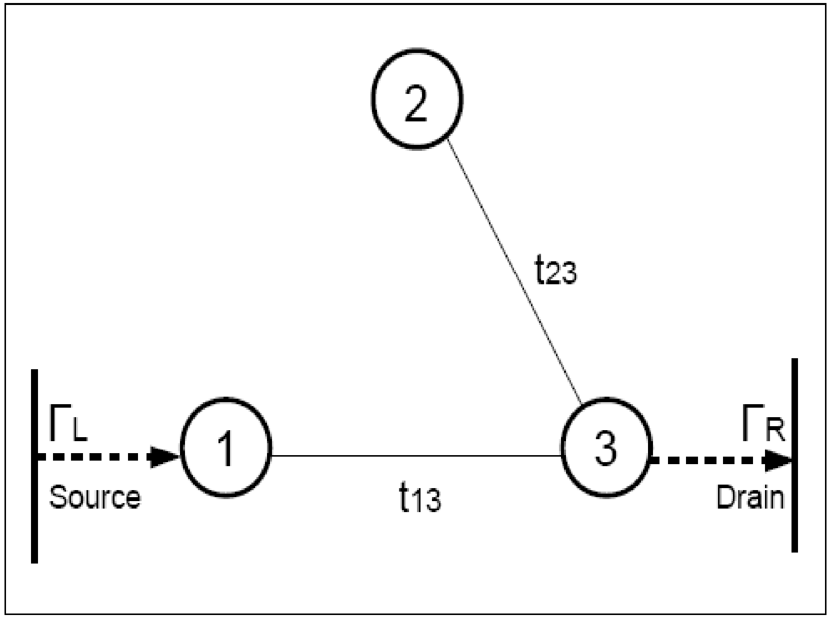
M. Busl et al., PRB (RC) 2010

Other TQDs configurations…..

Current Rectification

Double occupation just in QD 2

$$B_{ac} = 0$$



$$\varepsilon_1 = \varepsilon_2 + U_2$$

$$t_{13} = t_{23}$$

In this configuration no SB

$$\Delta_1 = \Delta_2 = \Delta_3$$

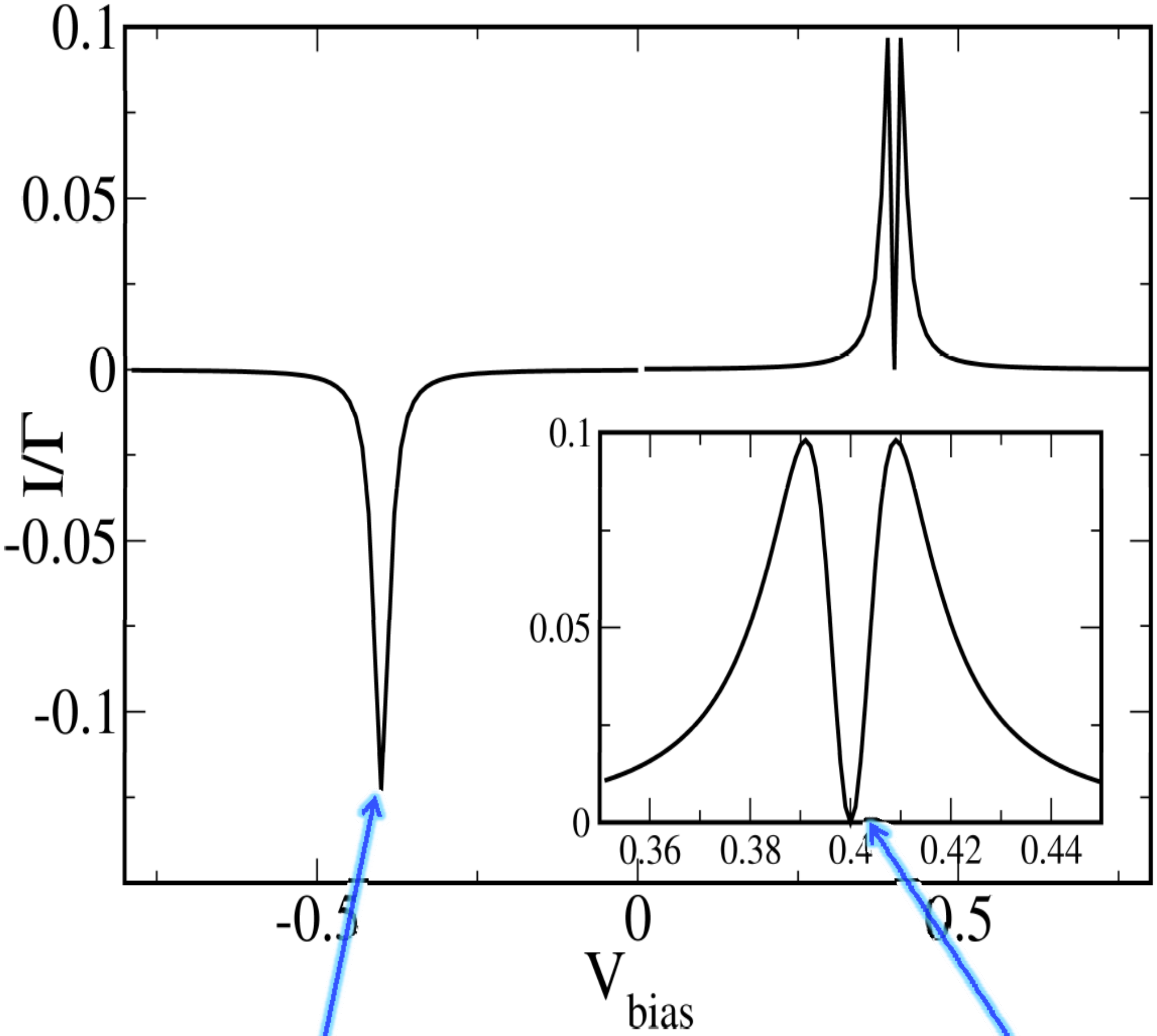
$$I(t) \propto \Gamma \rho_{3,3}(t)$$

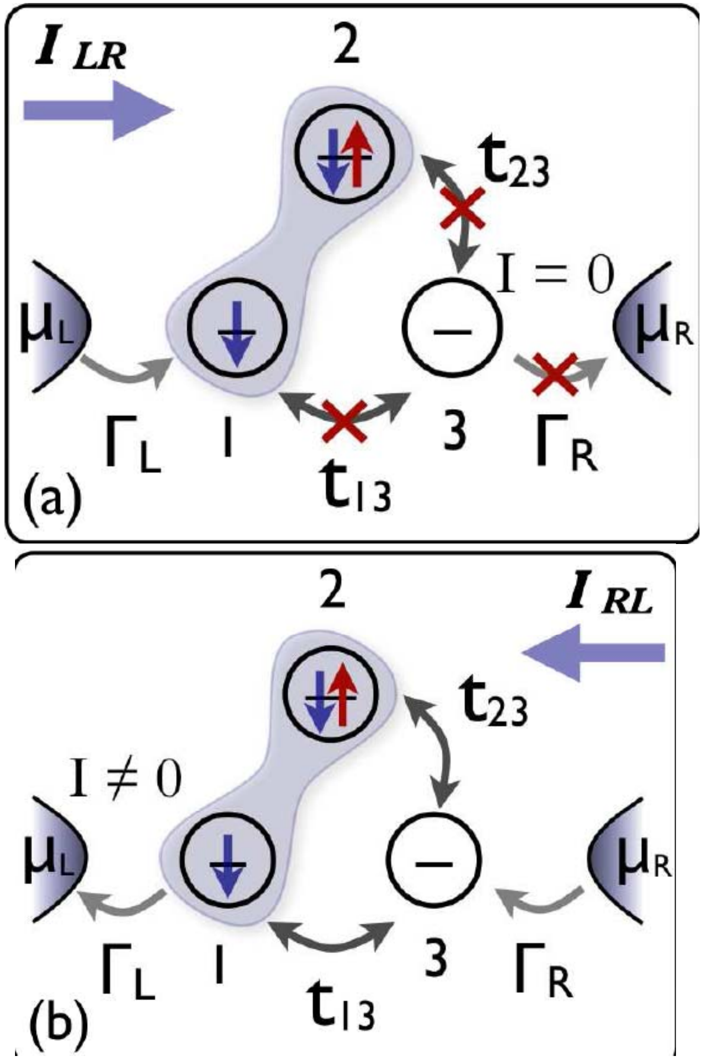
Dark State

$$|\Psi\rangle = \frac{1}{\sqrt{3}}\left(|\uparrow,\downarrow,0\rangle - |\downarrow,\uparrow,0\rangle + |0,\uparrow\downarrow,0\rangle\right)$$

Singlet state

Eigenstate of DM with eigenvalue 1
Eigenstate of the closed system





There is not trapping: Electrons tunnel from dot 1 to the left contact

Trapping occurs for positive bias voltage

Dark state: Singlet state

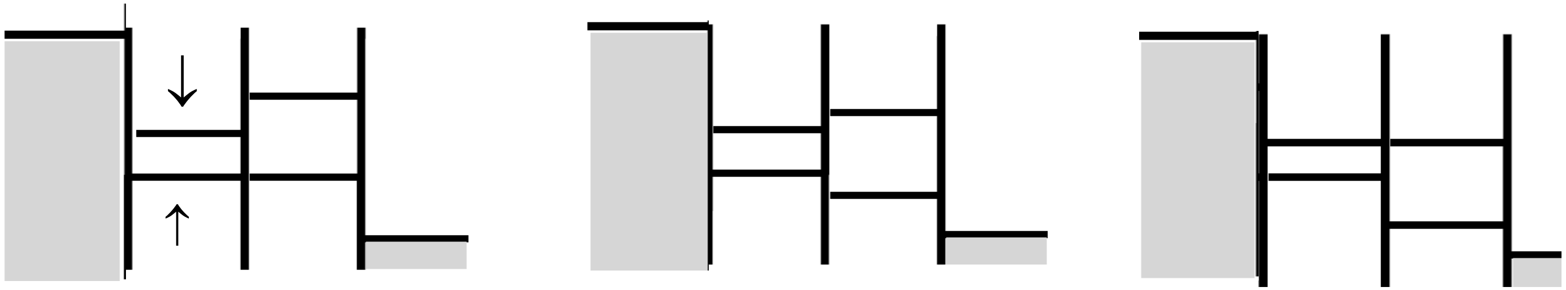
$$|\Psi\rangle = \frac{1}{\sqrt{3}}\left(|\uparrow,\downarrow,0\rangle - |\downarrow,\uparrow,0\rangle + |0,\uparrow\downarrow,0\rangle\right)$$

Spin filters and inverters

AlGaAs-InGaAs-AlGaAs-GaAs-AlGaAs

g factors engineering

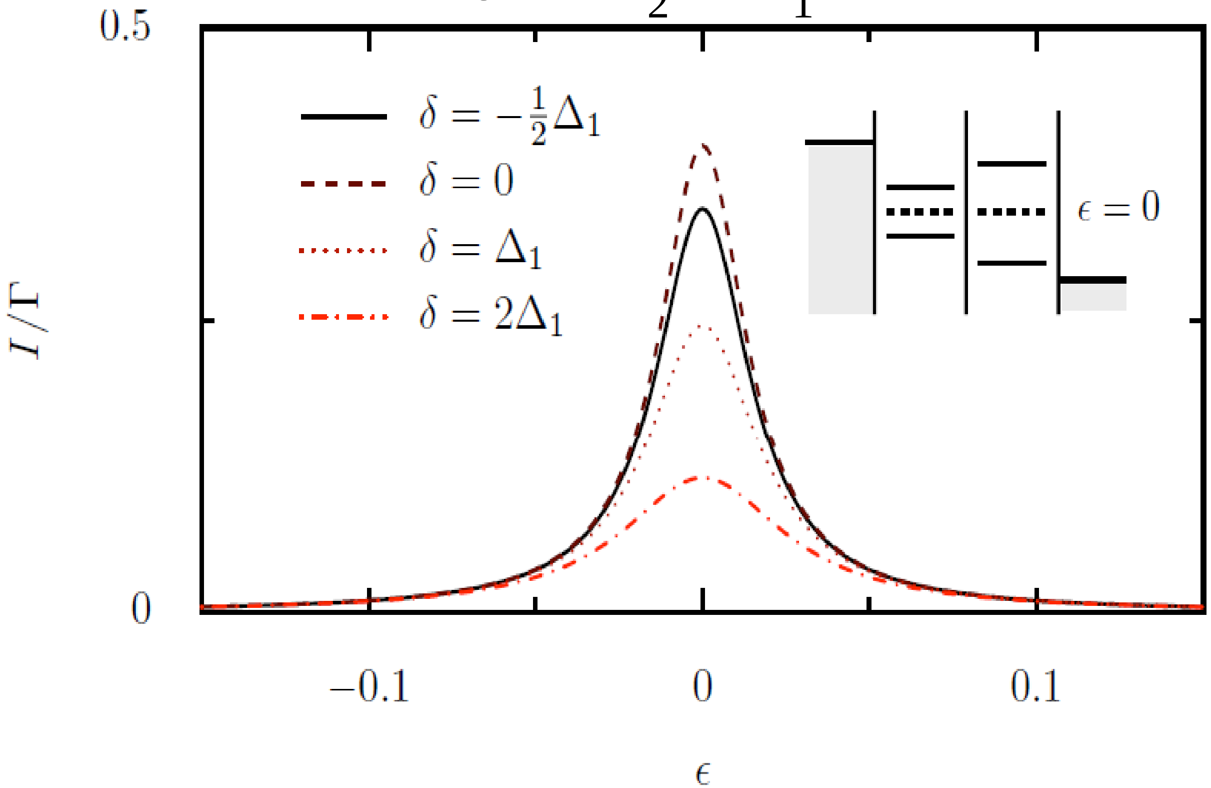
Spin Bottleneck | **Maximal current** | **Spin Bottleneck**



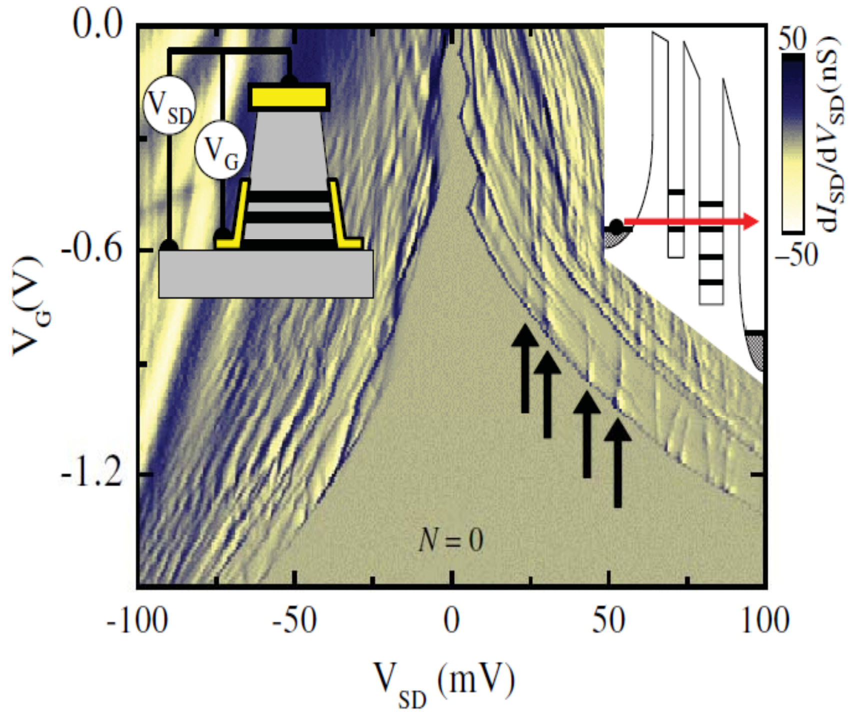
Bdc ≠0

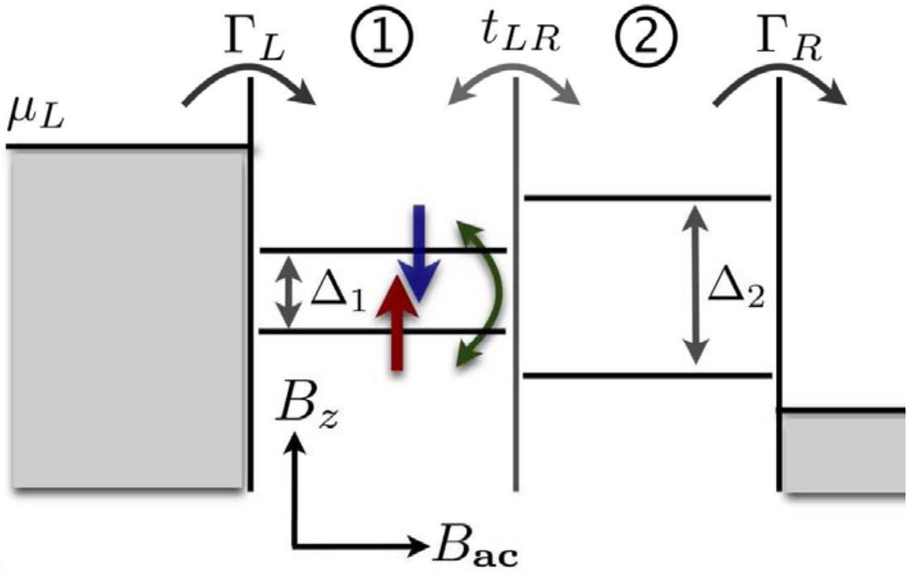
one electron

$$\delta = \Delta_2 - \Delta_1$$



S.M. Huang et al., PRL 2010





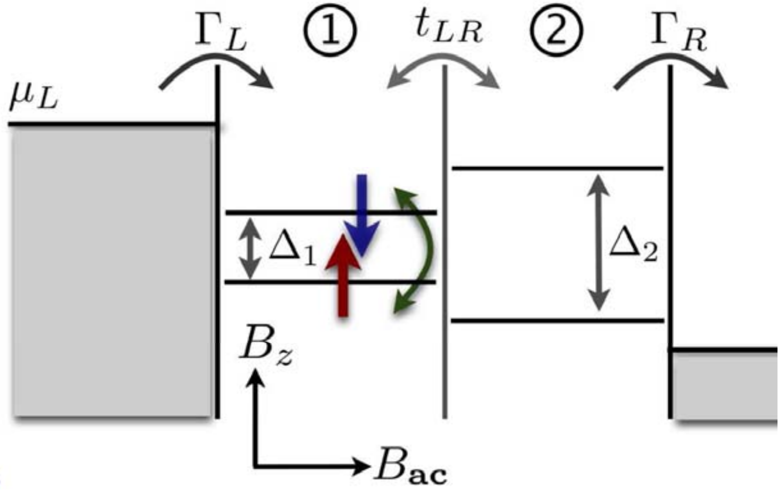
$$H^0_{\mathrm{B}}(t) = \sum_{i=1,2}[\Delta_i S_{zi} + B_{\mathrm{ac}}(\cos(\omega t)S_{xi} + \sin(\omega t)S_{yi})]$$

$$\hat{U}(t) = e^{-i\sum_j \omega_j \hat{S}_z^j t}$$

$$H^0 = \begin{pmatrix} -\frac{\Delta_1}{2} + \frac{\omega}{2} & \frac{B_{\mathrm{ac}}}{2} & -t_{\mathrm{LR}} & 0 \\ \frac{B_{\mathrm{ac}}}{2} & \frac{\Delta_1}{2} - \frac{\omega}{2} & 0 & -t_{\mathrm{LR}} \\ -t_{\mathrm{LR}} & 0 & -\frac{\Delta_2}{2} + \frac{\omega}{2} - \epsilon & \frac{B_{\mathrm{ac}}}{2} \\ 0 & -t_{\mathrm{LR}} & \frac{B_{\mathrm{ac}}}{2} & \frac{\Delta_2}{2} - \frac{\omega}{2} - \epsilon \end{pmatrix}$$

$$Resonance\ condition: \ \omega = \Delta_1$$

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}(|\uparrow_1\rangle \pm |\downarrow_1\rangle)$$



2 quasi-degenerate levels which differ in Bac

$$\delta = \Delta_2 - \Delta_1$$

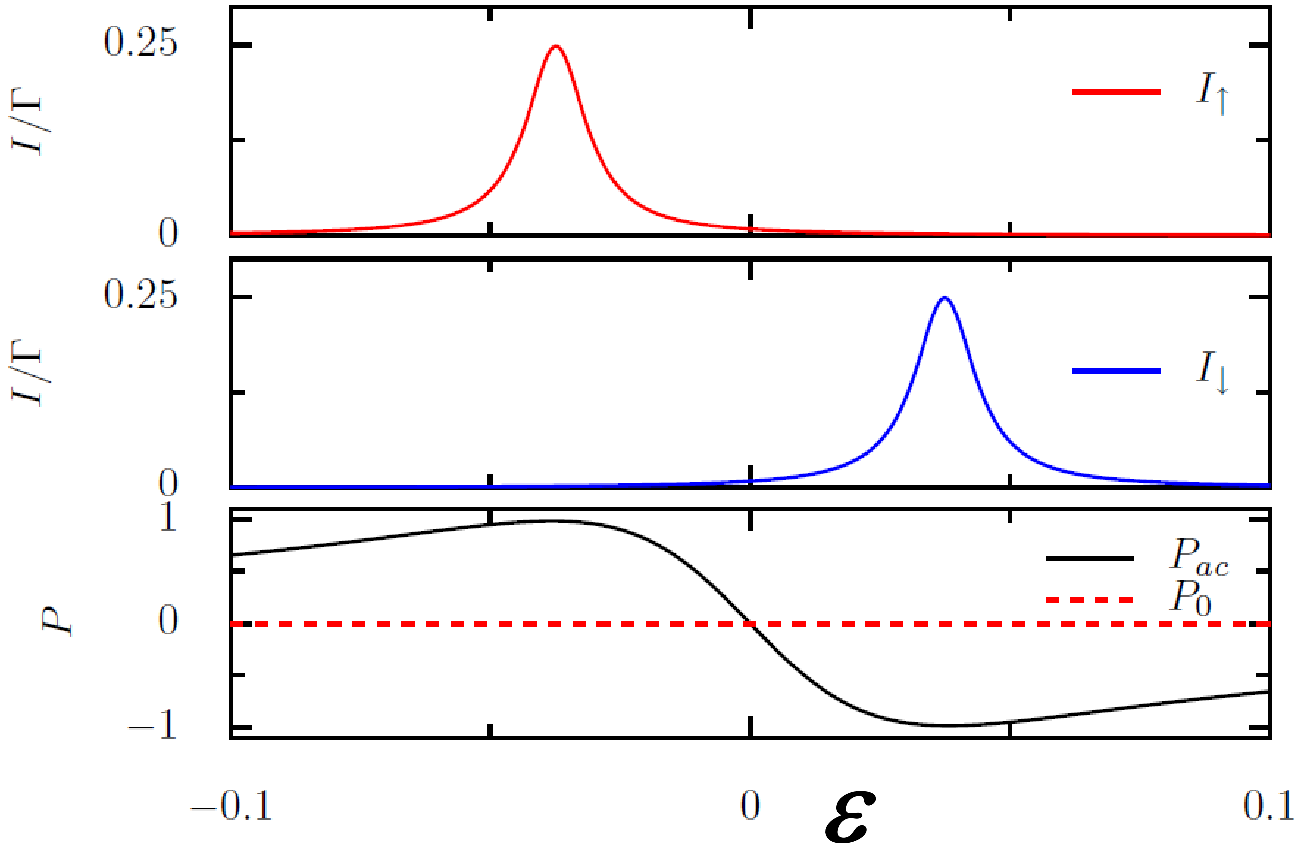
$$|\psi_2\rangle^+ = \frac{1}{N^+}\left(-\frac{\delta + \sqrt{B_{ac}^2 + \delta^2}}{B_{ac}}|\uparrow_2\rangle + |\downarrow_2\rangle\right)$$

$$|\psi_2\rangle^- = \frac{1}{N^-}\left(-\frac{\delta - \sqrt{B_{ac}^2 + \delta^2}}{B_{ac}}|\uparrow_2\rangle + |\downarrow_2\rangle\right)$$

at $B_{ac} \ll \delta$

$$|\psi_2\rangle^+ \rightarrow |\uparrow_2\rangle$$

$$|\psi_2\rangle^- \rightarrow |\downarrow_2\rangle$$

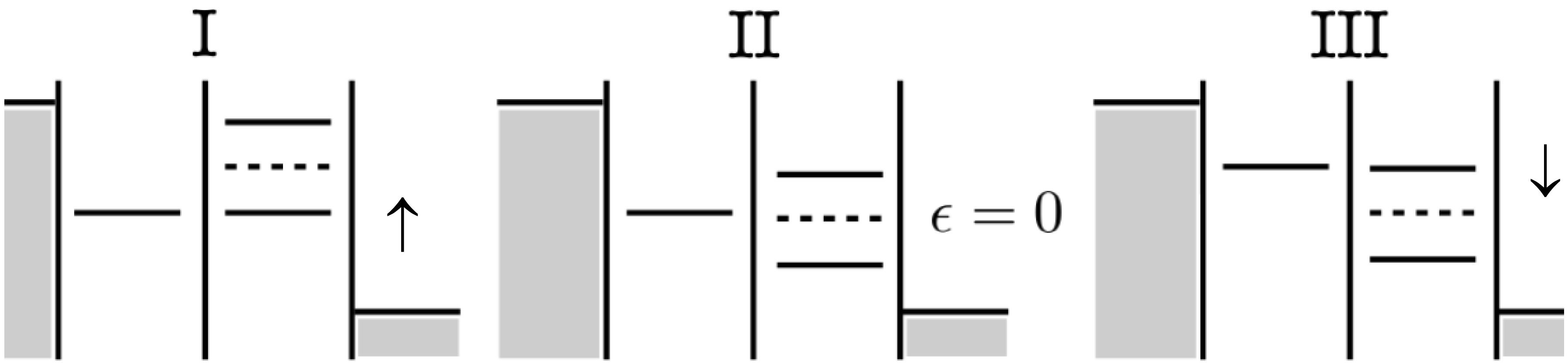


$$\Delta_1 = \omega_{AC}$$

Peaks at $\varepsilon = \pm\frac{\delta}{2}$

DQD as spin filter

Fully spin polarized current with opposite polarization at different detunings

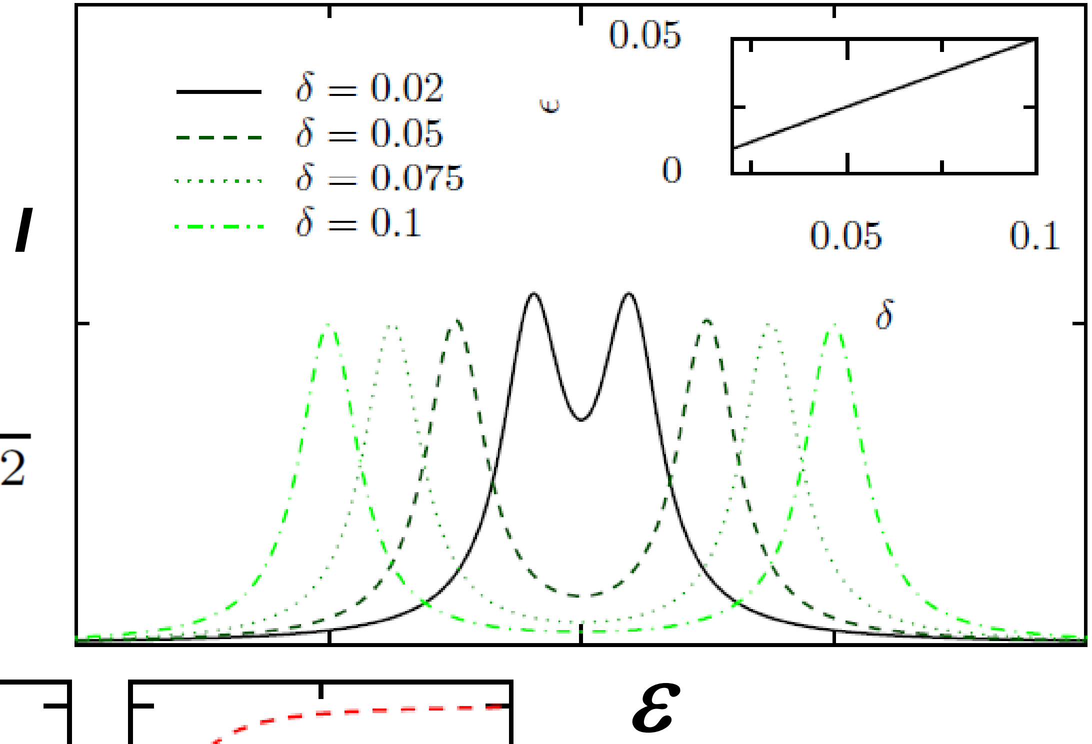


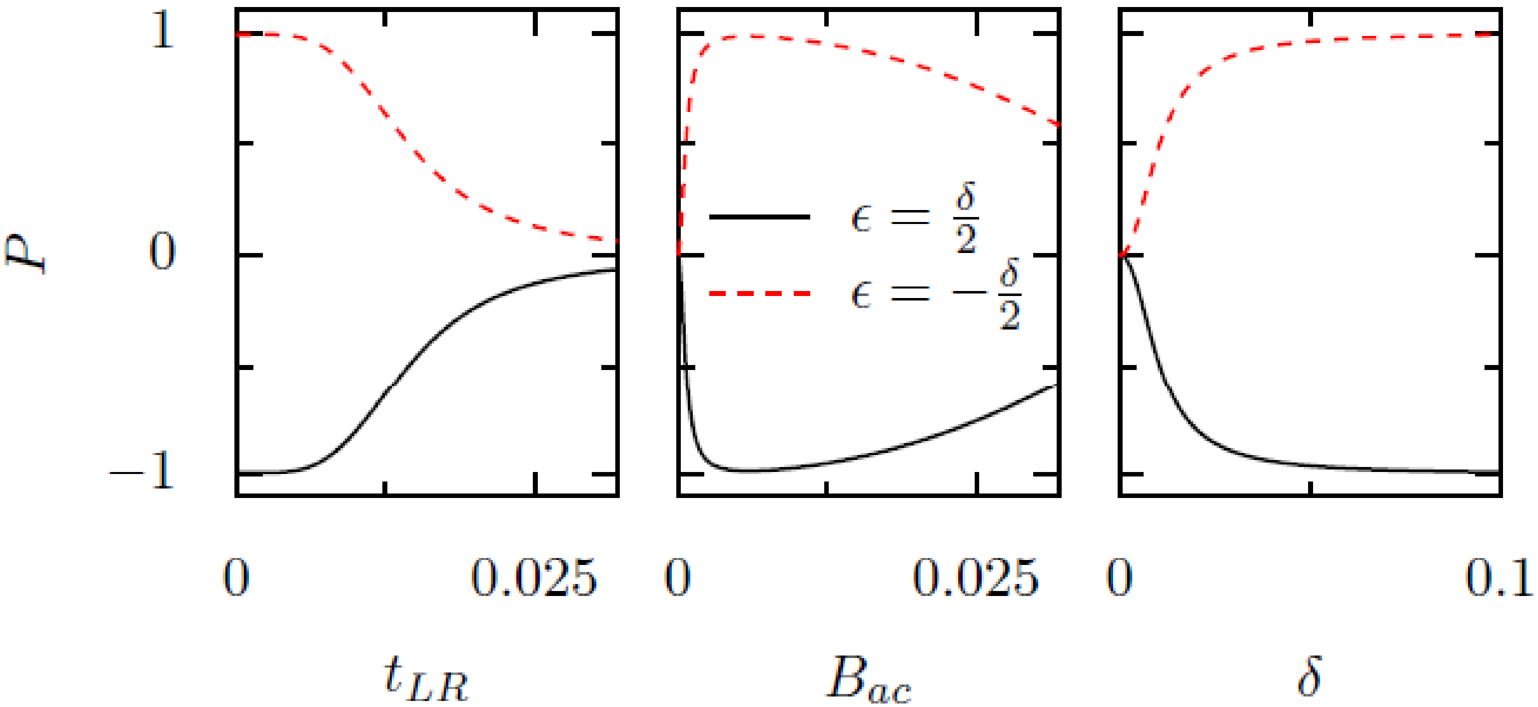
Levels are renormalized by the BAC

Resonance condition: $\omega = \Delta_1$

$$\lim_{t_{\mathrm{LR}}\to\infty} \frac{I_{\uparrow,\downarrow}}{e\Gamma} = \frac{1}{5}$$

$$\lim_{B_{\mathrm{ac}}\to\infty} \frac{I_{\uparrow,\downarrow}}{e\Gamma} = \frac{2t_{\mathrm{LR}}^2}{\Gamma^2 + 10t_{\mathrm{LR}}^2 + 4\epsilon^2}$$

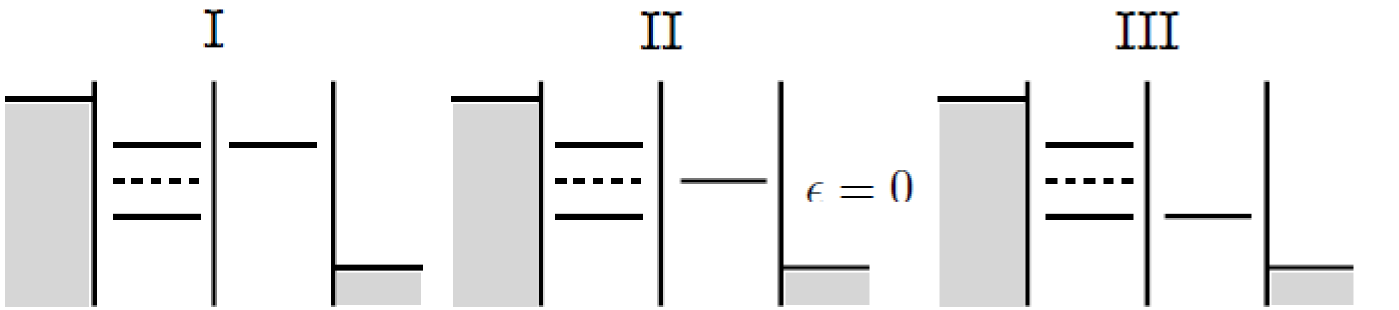




$$Resonance\ condition:\ \omega = \Delta_2$$

$$|\psi_2\rangle = \frac{1}{\sqrt{2}}(|\uparrow_2\rangle \pm |\downarrow_2\rangle)$$

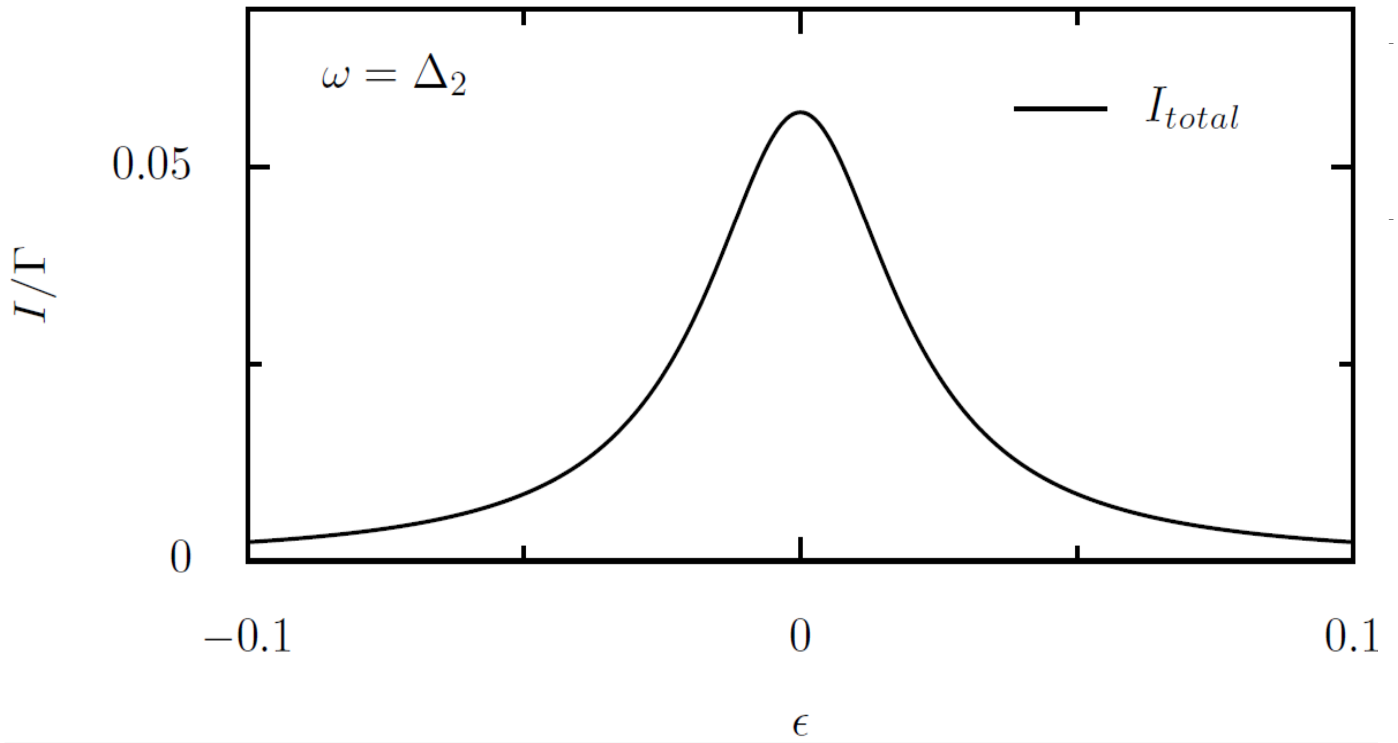
Spin bottleneck at



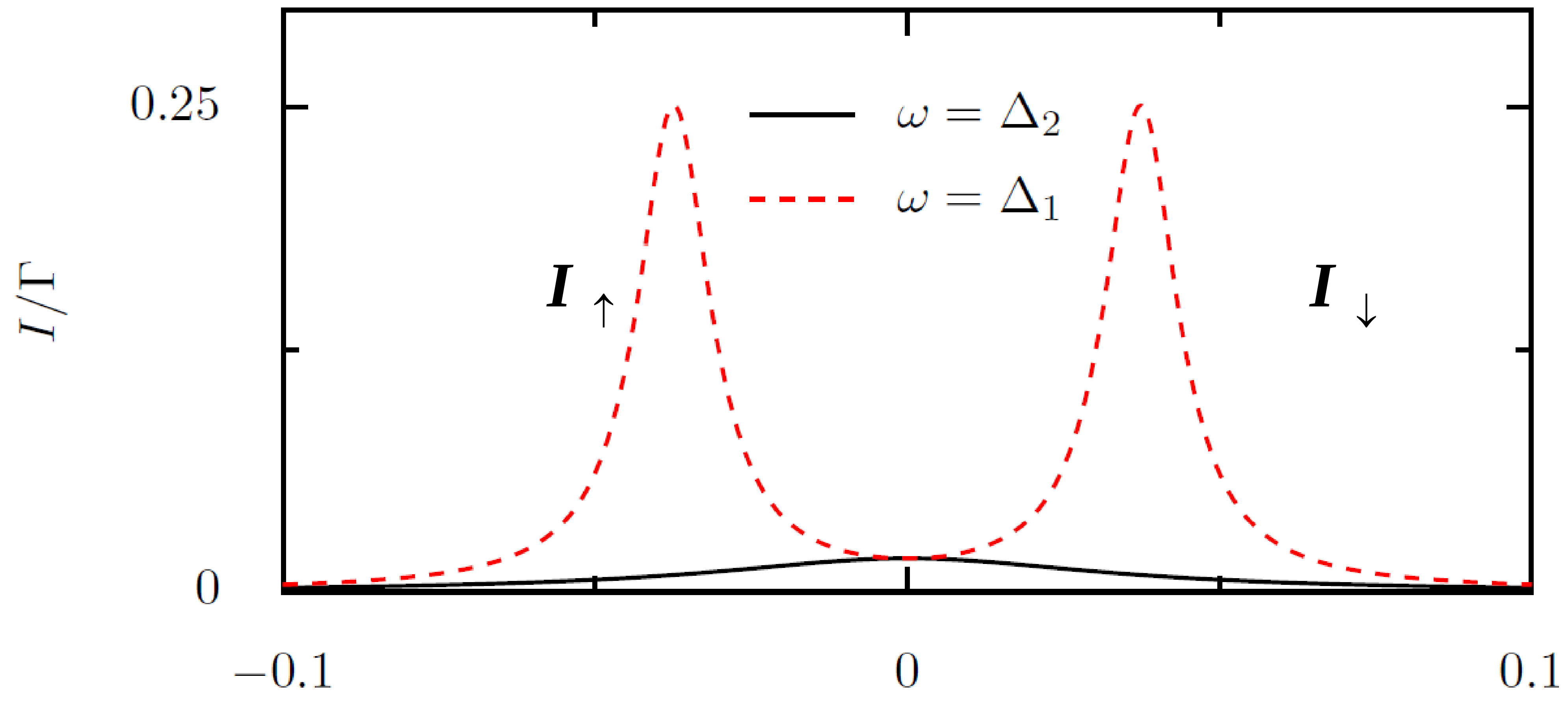
$$\varepsilon = \pm\frac{\delta}{2}$$

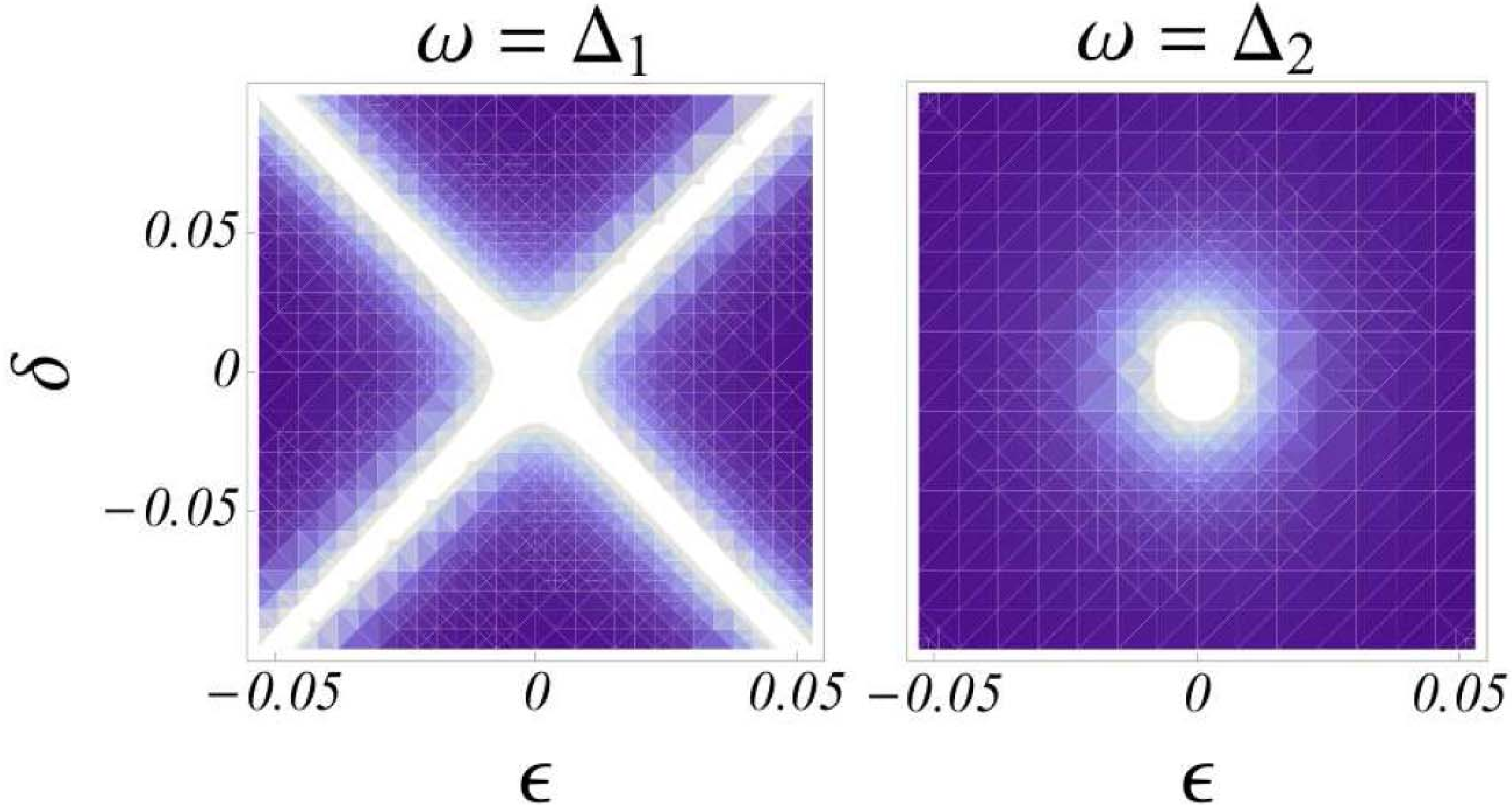
Strong spin level mixing in the drain dot: unpolarized current at:

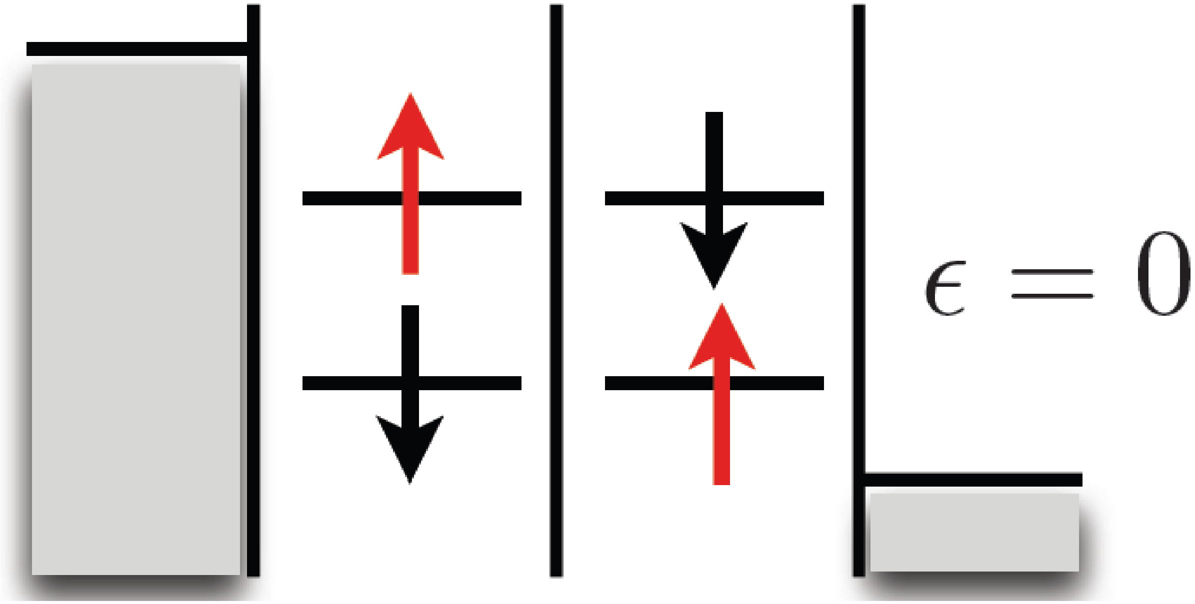
$$\varepsilon = 0$$



DQD as Spin Filter by tuning ω and Vgate



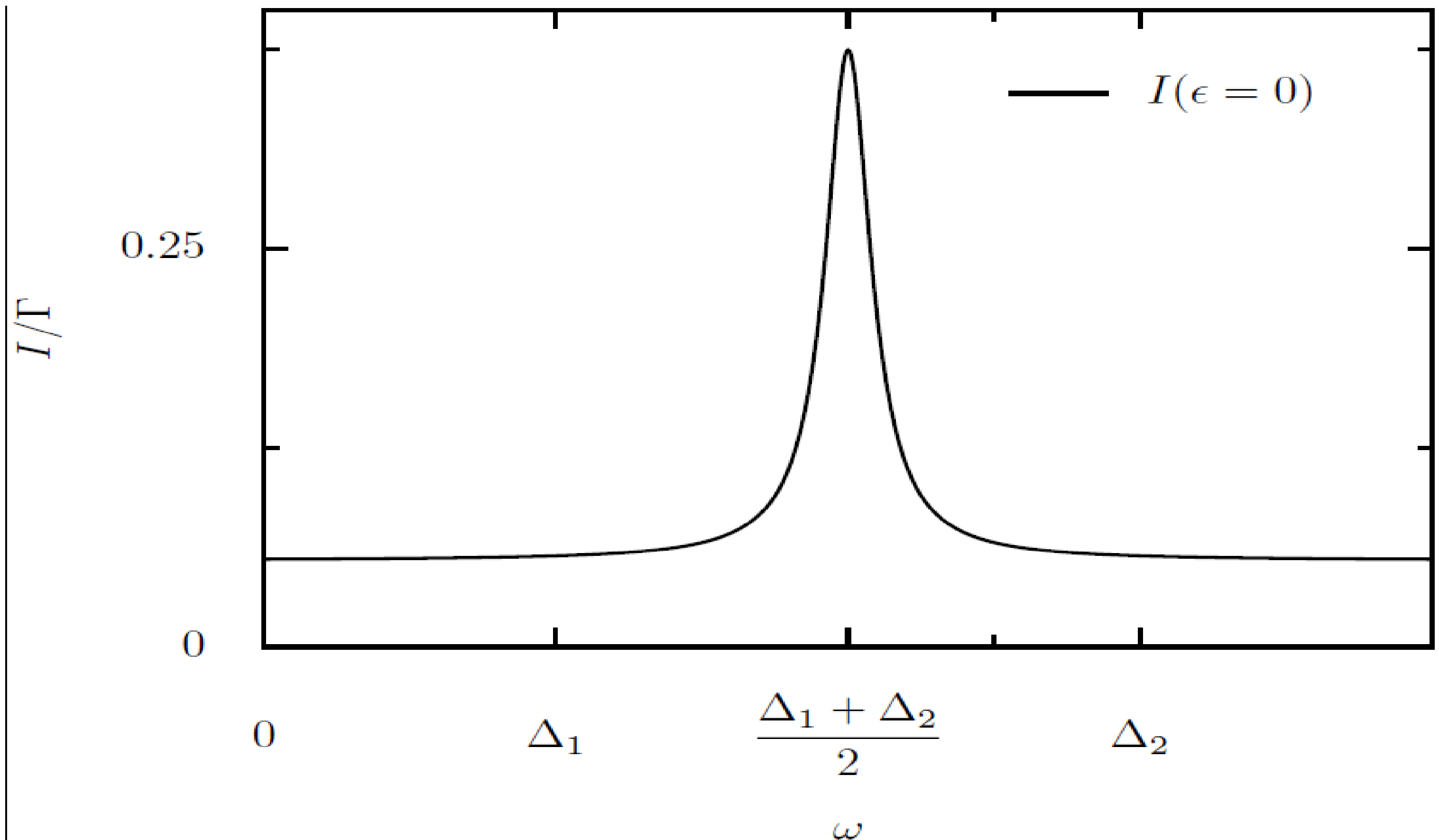

ω = Δ1
ω = Δ2
δ
0.05
0
−0.05
−0.05
0
0.05
ϵ
−0.05
0
0.05
ϵ



$$\omega = \frac{\Delta_1 + \Delta_2}{2}$$

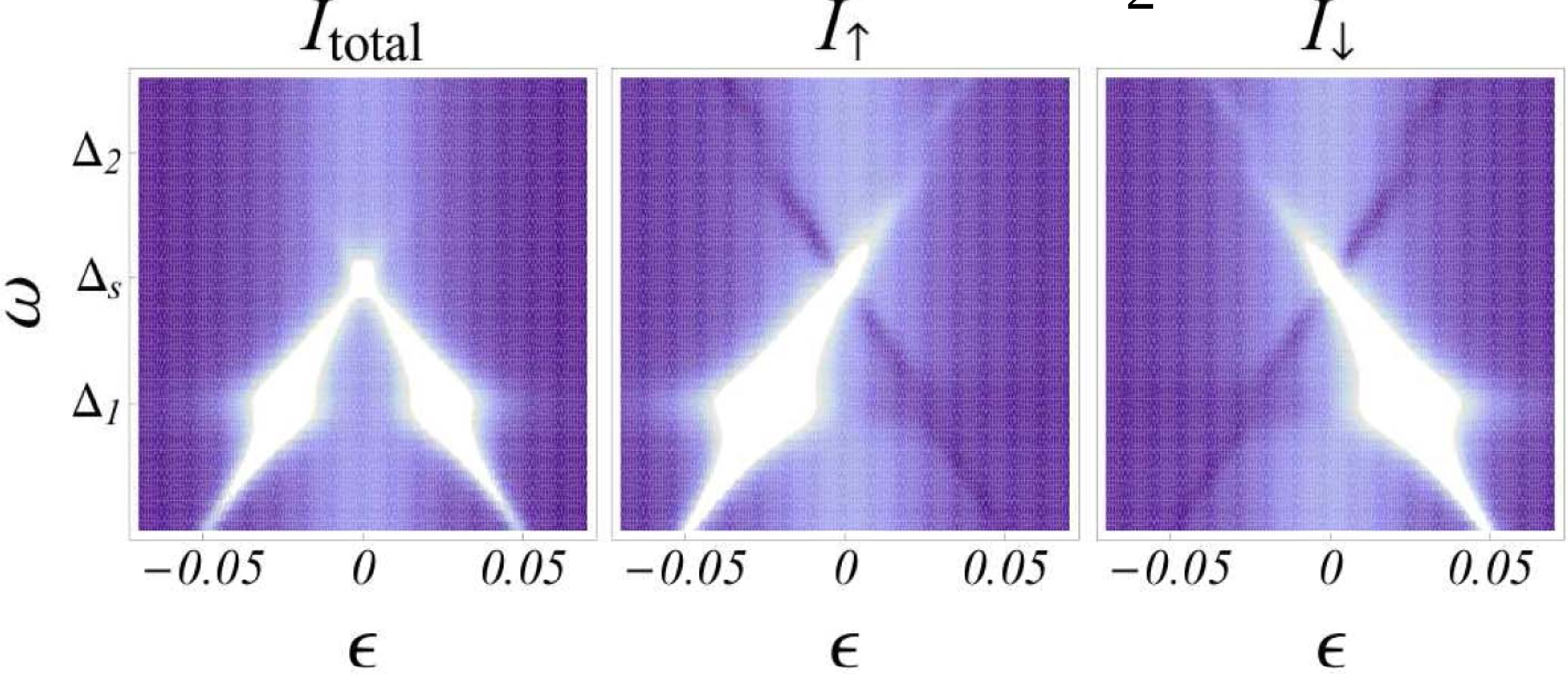
$$\Delta_1^* = \frac{\Delta_1 - \Delta_2}{2}$$

$$\Delta_2^* = -\Delta_1^*$$



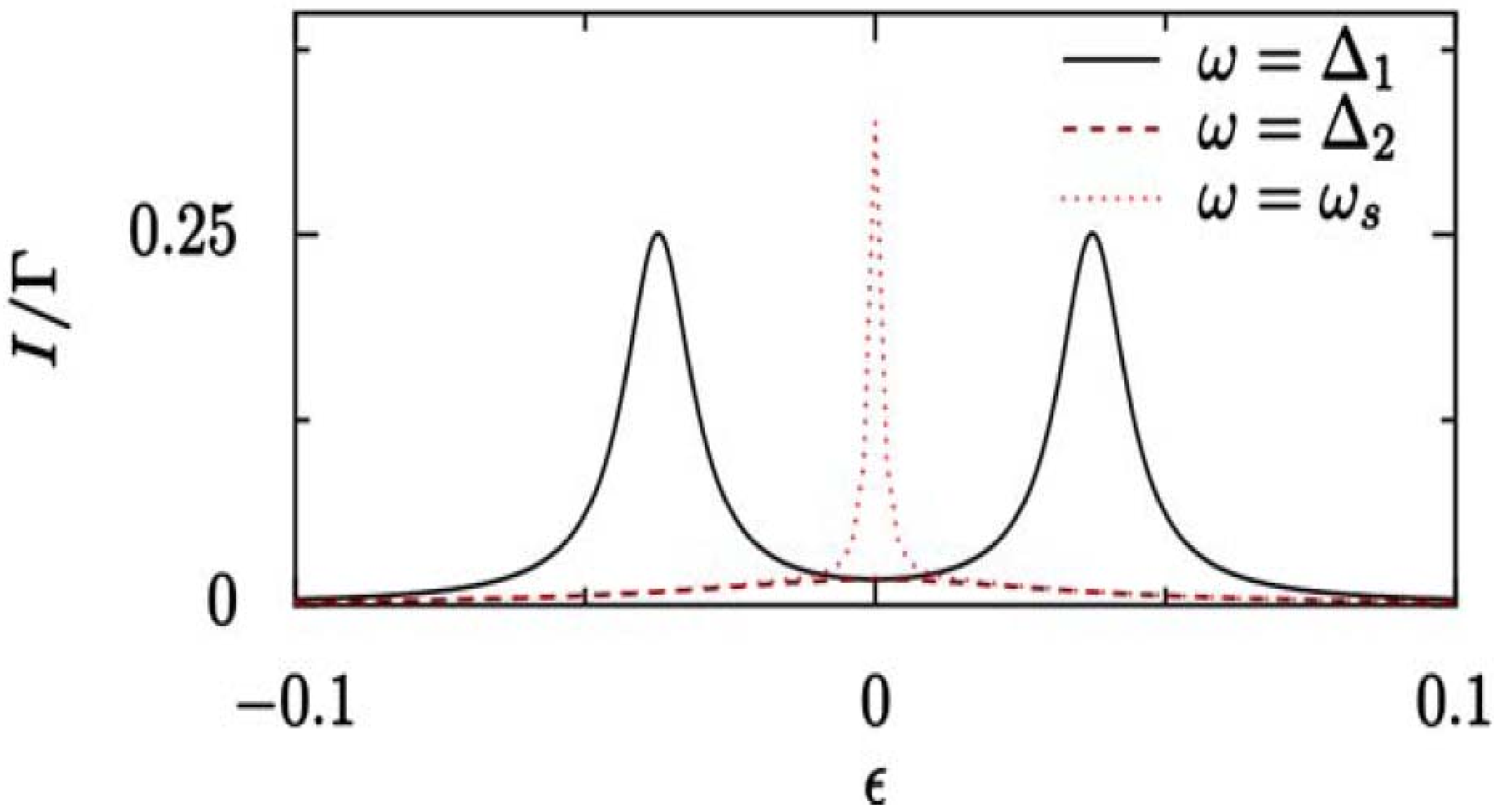
$$\Delta_2 > \Delta_1$$

$$\Delta_s = \frac{\Delta_1 + \Delta_2}{2} \qquad \Delta_2 > \Delta_1$$



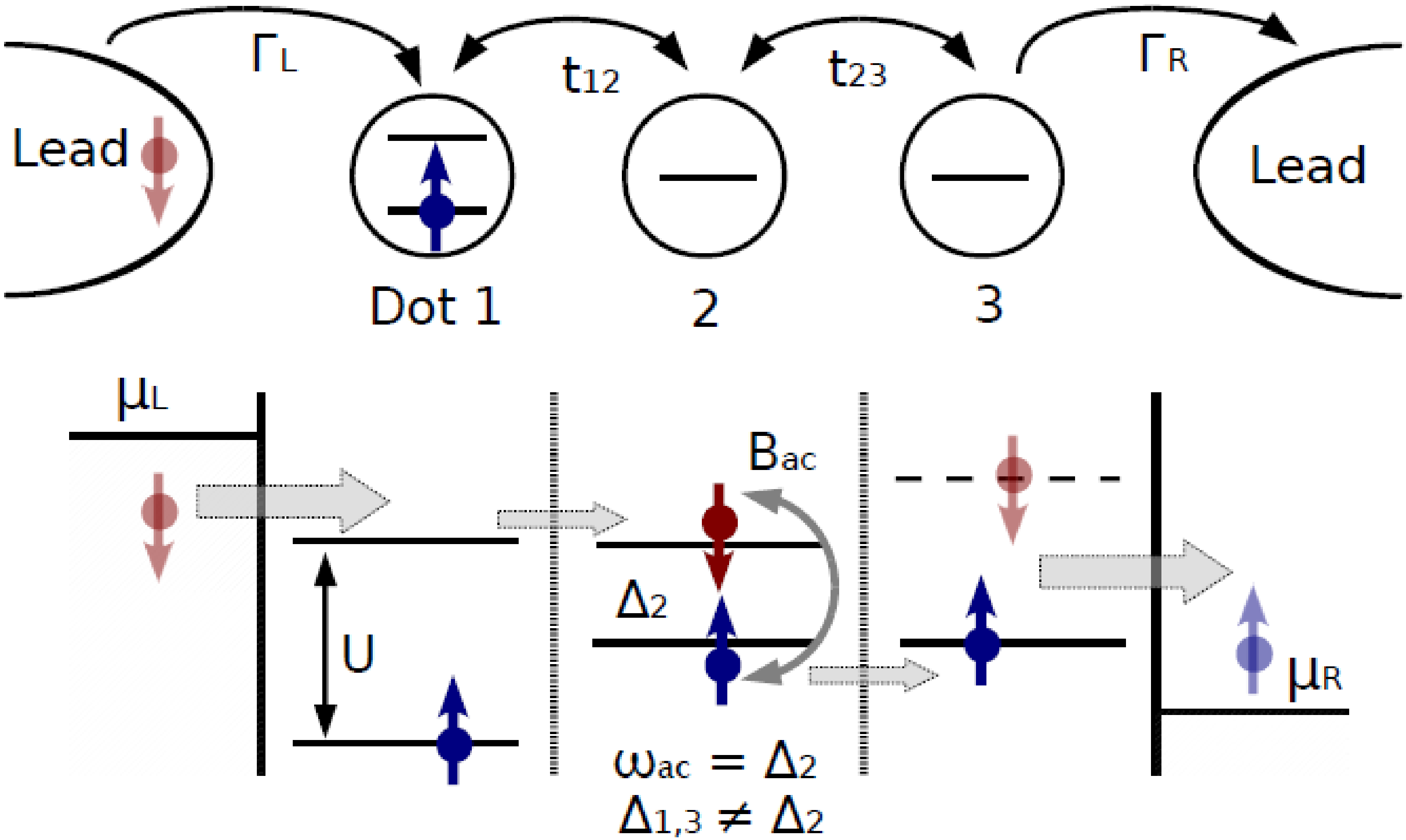
In resonance: $I = f(\delta) \quad \delta = \Delta_2 - \Delta_1$

Off resonance $I = f(\Delta_1, \Delta_2) \quad \Delta^*_{1,2} = \Delta_{1,2} - \omega$



$\omega = \Delta_1$
$\omega = \Delta_2$
$\omega = \omega_s$
0.25
I/Γ
0
−0.1
0
0.1
ϵ

ESR in TQD in series: from spin filter to spin inverter

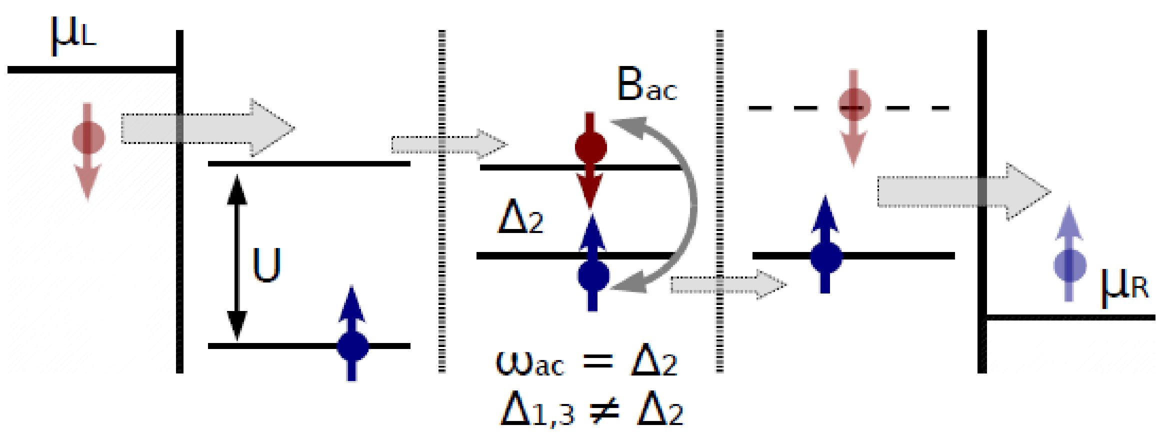


Double occupation allowed just in the left dot

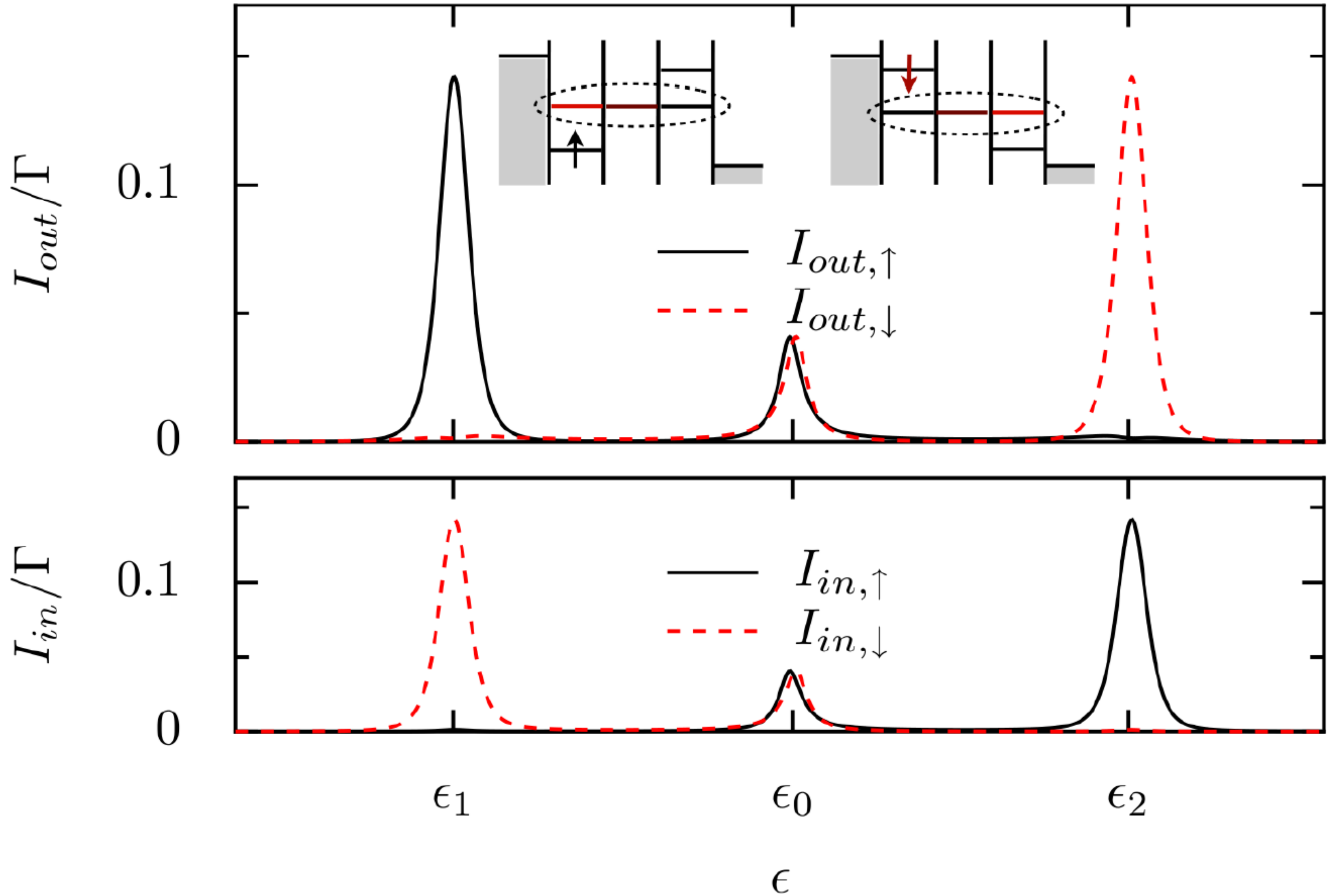
$$\Delta_L = \Delta_R = 0.13\boldsymbol{meV}$$

$$\Delta_C = 0.026\boldsymbol{meV}$$

TQD as spin inverter:

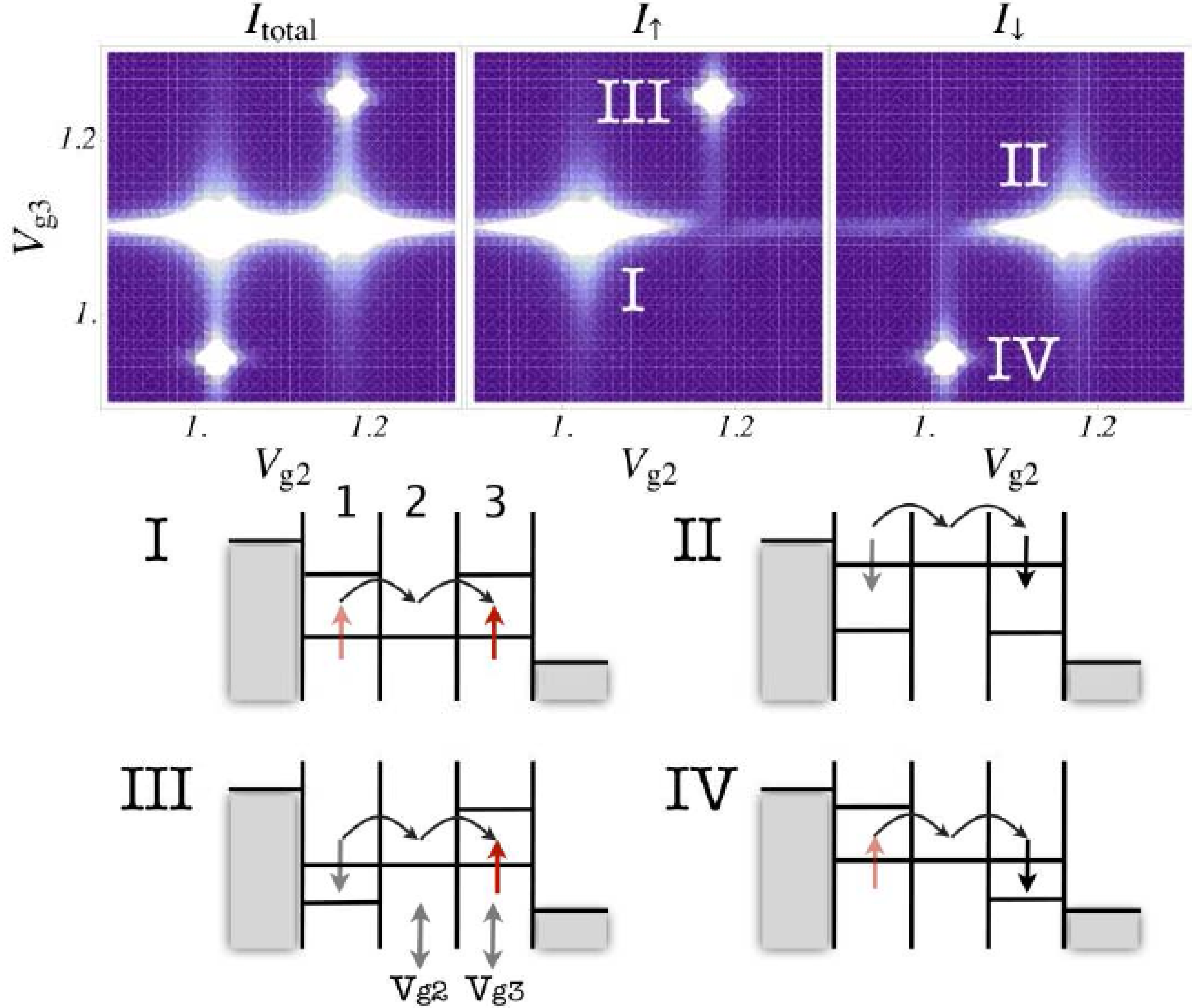


$$I_{\downarrow}^{in} \rightarrow I_{\uparrow}^{out}$$



$$I_{\uparrow}^{in} \rightarrow I_{\downarrow}^{out}$$

M. Busl et al., Phys. Rev. B, 82, 205304 (2010).



Conclusions

DQD's and TQD's $B_{dc} \neq 0, \quad B_{ac} \neq 0$

Application as spin filters and spin inverters: Spin up or down polarized current tuned by electric gates:

DQD's as spin filters

TQD's also as spin inverters

TQD's in triangular configuration:

1 e: coherent oscillations of one electron spin

2 e's: Bac induces Spin Blockade in DQD's and TQD's , both in series and triangular configurations:

Bac a tool for tuning SB